Open queueing network with batch transfers and partly unreliable systems.

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Abstract

In this paper we consider open exponential network with batch arrival and batch service. Each system has two regimes of functioning. We established the criterion of geometric product form stationary distribution existence and offered algorithm of checking its execution.

1 Introduction

Nowadays network models with unreliable systems become actual to a marked degree. Herewith, one pay considerable attention to research systems, which may partly lose their capacity for work, for example when systems pass from a certain regime to another one. At this paper we researched open queueing network with batch transfers. Each system has two functional regimes. The first regime we call "main". The group of customers which got its service in the second regime is removed from the network. We consider that the second regime service is not qualitative. As a result we have a mathematical model of network with batch transfer and two functional regimes. Besides, at this model we avoid the situation, when the node requires for service a group, which size is more than the number of customers at the system. An intensity of service depends on the number of customers at node. Because of such model description we made it easier to research queueing network. At first, we avoided many technical difficulties. Secondly, we offered an effective algorithm of finding the geometric product form stationary distribution.

2 The single station model

We consider single station model with batch arrivals and batch service. We assume that customers batches arrive at the station according to a Poisson process at rate λ (name this flow "ordinary"). When system is empty there is an additional input Poisson flow at rate λ^*. System has two functional regimes. With the probability p_1 it choose the first (main) regime. And with the probability p_2 - the second regime. Service times of batches are independent exponentially distributed random variables with rate $n\mu_l$, where n - is a number of customers at node, $l = 1, 2$ - is a number of regime. Note, that if there are n customers at the system, we choose for service a group of random size from 1 to n with the probability $\frac{1}{n}$. Arrival and service processes are independent. Sizes of arriving customers batches and sizes of additional batches $\{Y_i\}$, $\{Y_i^*\}$ are independent non negative identically distributed random variables with random distribution functions of A, A^*, probability functions a, a^* and probability generating functions $\tilde{A}$, $\tilde{A}^*$ accordingly. Let $X(t)$ be the number of customers at the system at time t. Then $X(t)$ is a continuous-time Marcov chain. Intensities of its transition are:

$$\begin{cases} q(n, n+k) = \lambda a(k) + \lambda^* a(k)^* & (k \geq 1), \\ q(n, n-k) = p_1 \frac{1}{n} n\mu_1 + p_2 \frac{1}{n} n\mu_2 = p_1\mu_1 + p_2\mu_2 & (n \geq k \geq 1), \end{cases}$$

Considering time-reversed process we found ergodicity conditions of Marcov process $X(t)$ and criterion of geometric stationary distribution existence.

Theorem 1. *Under condition of ergodicity:*

$$\lambda m_A < p_1\mu_1 + p_2\mu_2,$$

$X(t)$ has stationary distribution $\{\pi(n) = (1-c)c^n,\ n = 0, 1, \dots\}$ if and only if inequalities

$$(p_1\mu_1 + p_2\mu_2)kc^k \geq \lambda^* c^k + \lambda \sum_{s=0}^{k-1} a(k-s)c^s,\ k = 1, 2, \dots,$$

hold, where

$$c = \frac{\lambda + \lambda^*}{\lambda + \lambda^* + p_1\mu_1 + p_2\mu_2} \in (0, 1).$$

Moreover, parameters of additional flow may me founded by means of following equations:

$$\lambda^* a^*(k) = \Big[(p_1\mu_1 + p_2\mu_2)kc^k - \lambda^* c^k - \lambda \sum_{s=0}^{k-1} a(k-s)c^s\Big],\ k = 1, 2, \dots$$

Note, that with such intensities of transition process $X(t)$ is quasi-reversible.

3 Network model

We consider queueing network with set of nods $J = \{1, 2, \dots, N\}$. Customs batches arrive at the network according to a Poisson process at rates λ_i for nod $i \in J$ accordingly (ordinary flows). When nod $i \in J$ is empty there is an additional input Poisson flow at rate λ_i^*. Each system has two functional regimes. With the probability $p_i(1)$ it choose the first (main) regime. And with the probability $p_i(2)$ - the second regime for nod $i \in J$ accordingly. Service times of batches are independent exponentially distributed random variables with rate $n_i\mu_{i,l}$, where n_i - is a number of customers at node $i \in J$, $l = 1, 2$ - is a number of regime. If there are n_i customers at the system, we choose a group of random size from 1 to n_i with the probability $\frac{1}{n_i}$, $i \in J$. After the finishing of service process the groups which got their service in the second regime are removed from the network immediately, because we consider, that the second regime service is not qualitative. Sizes of arriving ordinary and additional groups are independent non negative identically distributed random variables with random distribution functions of A_i, A_i^*, probability functions a_i, a_i^* and probability generating functions $\tilde{A}_i$, $\tilde{A}_i^*$ for nod $i \in J$ accordingly. After the finishing of service process at the first regime at nod $i \in J$ batch is routed to nod $j \in J$ with the probability $p_{i,j}$ and with the probability $p_{i,0}$ is removed from network. Arrival and service processes are independent. Let $X_i(t)$ be the number of customers at station $i \in J$ at time t and set $\{\mathbf{X}(t) = (X_1(t),\ X_2(t),\ \dots,\ X_N(t))\}$. Then $X(t)$ is a continuous-time Markov chain. To research the network we considered common rates of arriving ordinary batches of customers at node j, consists of m units $\gamma_j(m)$ accordingly as well as functions:

$$\hat{\gamma}_j(m) = \lambda_j a_j(m) + \sum_{i \in J} c_i^m p_i(1)\mu_{i,1}p_{i,j}, \qquad j \in J. \tag{1}$$

Denote: $\tilde{\Gamma}_i(z_i) = \sum\limits_{m=1}^{\infty} \hat{\gamma}_i(m)z_i^m$, $i \in J$. Herewith $\tilde{\Gamma}_i(1) = \sum\limits_{m=1}^{\infty} \gamma_i(m)$ are rates of the flow of main batches of customers at node i accordingly, $\tilde{\Gamma}_i^{-1}(1)\tilde{\Gamma}_i(z_i)$ — generating function of ordinary groups sizes distribution, $i \in J$. Like for the single station model, considering time-reversed process, one can get following results:

Theorem 2. *$\mathbf{X}(t)$ has stationary distribution $\{\pi(\mathbf{n}) = \prod\limits_{j \in J}(1-c_j)c_j^{n_j},\ \ \mathbf{n} \in \mathbb{Z}_+^N\}$ if and only if conditions of ergodicity hold:*

$$\tilde{\Gamma}_j'(1) < \mu_j, \qquad j \in J, \tag{2}$$

inequalities:

$$\mu_j k c_j^k \geq \lambda_j^* c_j^k + \Gamma_j(1)\sum_{s=0}^{k-1} \gamma_j(k-s)c_j^s, \qquad k = 1, 2, \dots,\ j \in J, \tag{3}$$

hold. Where

$$c_j = \frac{\tilde{\Gamma}_j(1) + \lambda_j^*}{\tilde{\Gamma}_j(1) + \lambda_j^* + \mu_{j,1}p_j(1) + \mu_{j,2}p_j(2)} \in (0;1), \quad j \in J, \tag{4}$$

Under conditions of theorem parameters of additional flow may be found from:

$$\lambda_j^* a_j^*(k) = \mu_j k c_j^k - \lambda_j^* c_j^k - \Gamma_j(1)\sum_{s=0}^{k-1}\gamma_j(k-s)c_j^s, \qquad k = 1, 2, \dots,\ j \in J. \tag{5}$$

4 Algorithm of finding the geometric product form stationary distribution.

We offer the next algorithm of finding and the checking of stationary distribution existence.
1. We find solutions $\hat{\gamma}_j,\ j \in J$ of the traffic equations system:

$$\hat{\gamma}_j = \lambda_j + \sum_{i\in J}\frac{\hat{\gamma}_i + \lambda_i^*}{\mu_{i,1}p_i(1) + \mu_{i,2}p_i(2)}\mu_{i,1}p_i(1)p_{i,j} \quad j \in J.$$

Note, one can prove, that this system has unique non-trivial solution.
2. Find $\tilde{\Gamma}_j(1) = \hat{\gamma}_j,\ j \in J$.
3. We check the conditions of ergodicity 2. If one of them does not hold at least, we conclude that geometric product form stationary distribution does not exist.
4. We find $c_j \in (0;1),\ j \in J$ by formulas 4.
5. Compute $\hat{\gamma}_j(m)$ by formulas 1.
6. We check inequalities 3. If they do not hold we conclude that geometric product form stationary distribution does not exist. If 3 holds we consider the following point of algorithm.
7. We compute parameters of additional flow by formulas 5. Solving the system of inequalities 3 for λ_j^*, we find domain of values of additional flow rates. This domain of values defines values $\lambda_j^*,\ \ j \in J$, when stationary distribution $\{\pi\}$ has geometric product form.
8. We write down solution for stationary distribution:

$$\{\pi(\mathbf{n}) = \prod_{j\in J}(1-c_j)c_j^{n_j}, \ \ \mathbf{n} \in \mathbb{Z}_+^N\}.$$

5 Conclusion

We considered an exponential open queueing network with batch transfers. An intensity of service depended on the number of customers at node. Each service system had two functional regimes. After the service at regime, which was not main, serviced group was removed from the network. As a result we obtained necessary and sufficient conditions of geometric product form stationary distribution existence and constructed an algorithm of stationary distribution finding.

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