

# A multi-objective genetic algorithm for the optimization of electrical transmission networks

F. Cadini  
Dipartimento di Energia  
Politecnico di Milano  
Via Ponzio 34/3, I-20133 Milano, Italy  
francesco.cadini@polimi.it

E. Zio and C. A. Petrescu  
Dipartimento di Energia  
Politecnico di Milano  
Via Ponzio 34/3, I-20133 Milano, Italy

## Abstract

A multi-objective genetic algorithm is devised to improve the reliability of power transmission by adding lines to an existing electrical network. The objectives driving the search for the optimal network expansion are the transmission reliability efficiency and the cost of the added transmission links. The approach is applied to an example of literature.

## 1. Introduction

In this work, a method is developed for identifying strategies of expansion of an existing electrical network in terms of new lines of connection to add for improving the reliability of its transmission service, while maintaining limited the investment cost. The typical large size of electrical networks involves a combinatorial number of potential solutions of new connections, so that classical optimization techniques become inapplicable. Instead, a Multi-Objective Genetic Algorithm (MOGA) is used, driven by the objectives of maximizing the network global reliability efficiency Zio (2007) and minimizing the cost of the added connections.

## 2. Global reliability efficiency of the IEEE RTS 96 electrical transmission network

The transmission network system IEEE (Institute of Electrical and Electronic Engineers) RTS (Reliability Test System) 96 is considered (Figure 1a) Billinton (1994). The network consists of 24 bus locations (numbered in bold in the Figure) connected by 34 lines and transformers. The transmission lines operate at two different voltage levels, 138 kV and 230 kV. The 230 kV system is the top part of Figure 1a), with 230/138 kV tie stations at Buses 11, 12 and 24.

Figure 1b) gives the graph representation of the network, which can be mathematically synthesized by the so-called adjacency matrix  $\{a_{ij}\}$ , a  $24 \times 24$  matrix whose entry is 1 if there is a line or transformer (hereafter called edge) between bus locations (hereafter called nodes)  $i$  and  $j$  and 0 otherwise. The entries on the diagonal elements  $a_{ii}$  are undefined, and set to 0 for convenience.

To capture the failure behavior of the network, the reliability of its connecting edges are included in the framework of analysis by means of the formalism of weighted networks, the weight  $w_{ij}$  associated to the edge between the pair of nodes  $i$  and  $j$  being its reliability:

$$p_{ij} = e^{-\lambda_{ij} \cdot T} \quad (1)$$

where  $\lambda_{ij}$  is the failure rate of edge  $ij$  linking nodes  $i$  and  $j$  and  $T$  is a reference time ( $T=1$  year, in this work). For the case study of the IEEE RTS 96 transmission network, the failure rates of the components (transmission lines and transformers) have been taken from Billinton (1994).

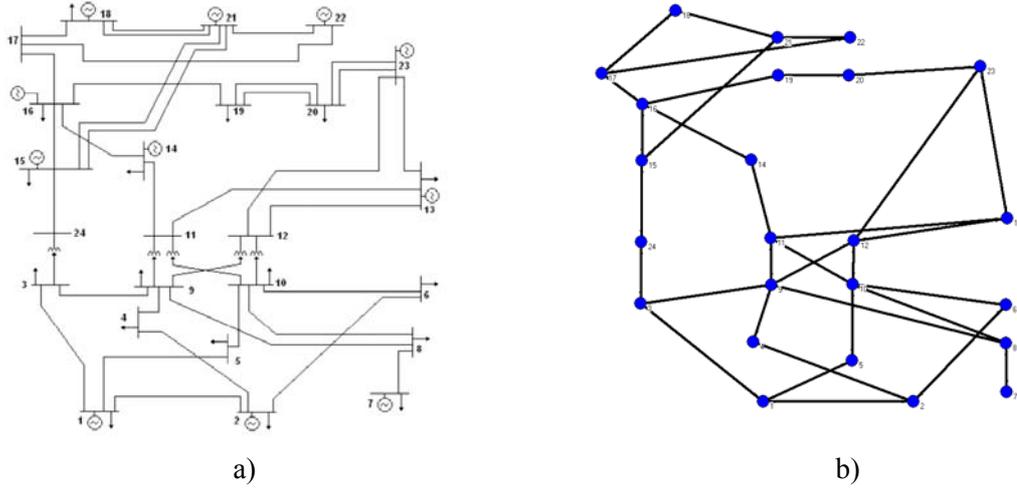


Fig. 1. a) IEEE RTS 96 transmission network; b) IEEE RTS 96 graph representation

On the basis of the adjacency and reliability matrices  $\{a_{ij}\}$  and  $\{p_{ij}\}$ , the matrix of the most reliable path lengths  $\{rd_{ij}\}$  can be computed Zio (2007). The reliability efficiency  $RE[G]$  of the graph  $G$  can then be defined Zio (2007):

$$RE[G] = \frac{1}{N(N-1)} \sum_{i,j \in N, i \neq j} (1/rd_{ij}) \quad (2)$$

For the IEEE RTS 96 network,  $RE = 0.2992$ , which is a relatively high value representative of a globally reliable network.

### 3. Multi-Objective optimization by Genetic Algorithms

Genetic Algorithms (GAs) are optimization methods aiming at finding the global optimum of a set of real objective functions,  $F \equiv \{f(\cdot)\}$ , of one or more decision variables,  $U \equiv \{u\}$ , possibly subject to various linear or non linear constraints. The terminology adopted in GAs contains many terms borrowed from biology, suitably redefined to fit the algorithmic context. Thus, GAs operate on a set of (artificial) chromosomes, which are strings of numbers, generally sequences of binary digits (bits) 0 and 1, coding the values of the decision variables. The values of the objective functions in correspondence of the values of the decision variables of a chromosome, give the fitness of that chromosome. The GA search is performed by constructing a sequence of populations of chromosomes, the individuals of each population being the children of those of the previous population and the parents of those of the successive population. The initial population is generated by randomly sampling the bits of all the strings; at each step in the search sequence, the new population is obtained by probabilistically manipulating the strings of the old population with fitness-improving rules which mimic genetic evolution. The search sequence continues until a pre-established optimality termination criterion is reached.

Typically, several possibly conflicting objective functions  $f_i(\cdot)$ ,  $i = 1, 2, \dots, n_f$ , must be evaluated in correspondence of each decision variable vector  $U$  in the search space. In this case, the GA search proceeds by comparing the solutions in terms of the concepts of *Pareto optimality and dominance* Goldberg (1989); the decision variable vectors which are not dominated by any other of a given set are called *nondominated* with respect to this set; the decision variable vectors that are nondominated within the entire search space are said to be *Pareto optimal* and constitute the so called *Pareto optimal front*, which is the object of the optimization.

#### 4. Optimal network expansion

A MOGA is constructed for identifying the best improvements in the connection structure of a network, aimed at increasing its global reliability of transmission at acceptable costs. The improvements are obtained by addition of new lines between nodes with no direct connection in the original network. Given the lack of geographical information on the nodes locations, for simplicity and with no loss of generality, three typologies of lines have been arbitrarily chosen as the minimum, the mean and the maximum values of the failure rates of the transmission lines taken from Billinton (1994):  $\lambda_1 = 0.2267$  outages/yr;  $\lambda_2 = 0.3740$  outages/yr;  $\lambda_3 = 0.5400$  outages/yr.

The addition of a new line requires an investment cost assumed inversely proportional to the failure rate. The network cost can be then defined as:

$$C[G] = \sum_{i,j \in N, i \neq j} (1/\lambda_{ij}) \quad (3)$$

The reliability cost of the original IEEE RTS 96 is  $C = 332.0120$  in arbitrary monetary units.

From the algorithmic point of view, a proposal of improvement amounts to changing from 0 to 1 the values of the elements in the adjacency matrix corresponding to the added connections. The only physical restriction for adding direct new connections is that the connected nodes must be at the same voltage level (138 or 230 kV), otherwise the addition of a transformer would also be needed. From the genetic algorithm point of view, the generation of proposals of network improvements can be achieved by manipulating a population of chromosomes, each one with a number of bits equal to 214 which is double the number of zeros (i.e., the number of missing direct connections  $ij$ ) in the upper triangular half of the symmetric adjacency matrix  $\{a_{ij}\}$ . The bits are dedicated to each missing direct connection  $ij$  so as to code the three different available types of lines with failure rates  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ : in other words, the bit-string (00) is used to code the absence of connection, (01) connection line with a  $\lambda_1$ -type line, (10) connection with a  $\lambda_2$ -type line and (11) connection with a  $\lambda_3$ -type line. The initial population of 200 individuals is created by uniformly sampling the binary bit values.

Figure 2 shows the Pareto dominance front (squares) obtained by the MOGA at convergence after  $10^3$  generations; the circle represents the original network with  $RE = 0.2992$  and  $C = 332.0120$ , while the star represents the network fully connected by the most reliable transmission lines  $\lambda_1 = 0.2267$  occ/yr, for which  $RE = 0.57$  and  $C = 804.1072$ .

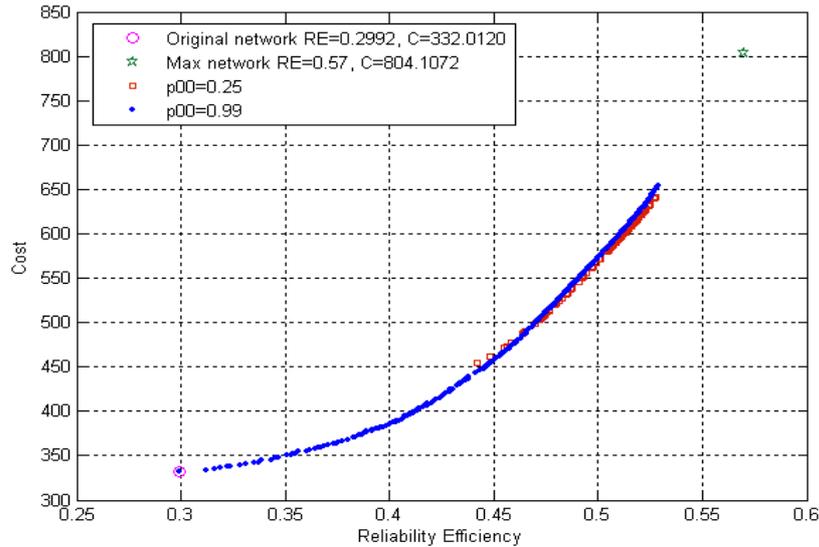


Fig. 2. Pareto front reached by the MOGA search procedures considered

The optimality search is biased from the beginning (from the initial population) towards highly connected network solutions, because the string (00) has a probability of 0.25 whereas the probability

of adding a connection of any one of the three available types (i.e., the probability of the strings 01, 10, 11) is 0.75; this drives the population evolution to highly connected networks in the Pareto front (squares in Fig. 2), all with values  $RE \geq 0.4417$ ,  $C \geq 454.4738$  and numbers of added connections exceeding 60.

In practical applications only a limited number of lines can be added, due to the large investment costs and other physical constraints. To overcome the bias in the search towards the addition of connections, the procedure of creation of the initial population is modified to favor the absence of connections (with probability 0.99). The effect of favoring few added connections in the initial population is that of reaching a Pareto front which is more concentrated on low cost networks, characterized by a limited number of added connections (dots in Fig. 2).

Table 1 lists the five solutions of lowest cost identified by the MOGA search with  $p\{00\} = 0.99$ : the added connections improve the network global reliability efficiency with small costs; the new links are located exclusively in the 230 kV transmission system (top part of Figure 1 a)), which is characterized by less reliable connections than the 138 kV transmission system; nodes 11 or 12 appear in all the new connections proposed, as they are central for power transmission.

Table 1. The five solutions of lowest cost obtained by the MOGA search with  $p\{00\} = 0.99$

p{00} = 0.99; p{10} = 0.0033; p{01} = 0.0033; p{11} = 0.0033.				
No of lines	Line-type	Connected nodes	Reliability Efficiency	Cost
1	$\lambda_3$	(11,21)	0.3120	333.9
1	$\lambda_2$	(12,21)	0.3162	334.7
1	$\lambda_1$	(12,21)	0.3190	336.4
2	$\lambda_2$	(11,16)	0.3192	336.5
	$\lambda_3$	(11,18)		
2	$\lambda_2$	(11,16)	0.3223	337.4
	$\lambda_2$	(12,17)		

## 5. Conclusions

A MOGA for improving an existing electrical transmission network has been implemented with the objective of identifying the location and type of lines to be added for maximizing the network transmission reliability, while maintaining the investment costs limited. A procedure in which the initial population favours the non-addition of lines ( $p\{00\} = 0.99$ ) has been implemented for individuating realistic network expansion solutions made of few new transmission lines.

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## References

- [1] Zio, E. (2007): From Complexity Science to Reliability Efficiency: A New Way of Looking at Complex Network Systems and Critical Infrastructures. *Int. J. Critical Infrastructures*, Vol. 3, Nos. 3/4, pp. 488--508.
- [2] Billinton, R., Li, W. (1994): Reliability Assessment of Electric Power Systems Using Monte Carlo Methods, pp.229-308.
- [3] Latora, V., Marchiori, M. (2001): Efficient Behavior of Small-World Networks. *Physical Review Letters*, Vol. 87, N. 19.
- [4] Goldberg, D.E. (1989): "Genetic algorithms in search, optimization, and machine learning", Addison-Wesley Publ. Co.