Modeling Lifetime Distribution under Variable Environment and Its Applications

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Abstract

Systems are often operated under variable environment. In this case, the failure probability of the system depends on the level of stress which is induced by the operational environment. In order to describe the failure process of the system under variable environment, it is essential to model lifetimes under different stress levels and connect them reasonably. In this paper, a general method for modeling lifetime distributions under variable environment will be presented. Its applications to accelerated burn-in model, and to general standby system will be introduced.

1 Introduction

The environment under which a system is operated is generally not static and in many cases it varies time to time. Higher stress causes accelerated deterioration, which results in earlier system failure. In this sense, it can be roughly said that the lifetime under severer environment is *shorter* than that under milder environment. Here, the precise meaning of 'shorter lifetime' should be stated in an appropriately defined probabilistic sense.

In practice, there could be many different situations where the change of environments or stress levels under which the system is operated occurs. For instance, in accelerated burn-in procedure, a system is tested under severer environment during burn-in and it is operated under milder environment during field operation. For a warm standby system, a standby unit works under a milder environment and then it is operated under severer one after it is activated.

In this paper, a general method for modelling lifetime distributions under variable environment will be presented. Its applications to accelerated burn-in model, to general standby system will be introduced. In Section 2, a probabilistic frame for modelling lifetime distributions under different stress levels, which is based on the basic statistical property commonly used in accelerated life tests, will be introduced. In Section 3, some of its applications will be illustrated. Finally, some concluding remarks will be given in Section 4.

2 Modelling Lifetime Distribution under Varying Environment

In this section, we consider the case when a system is operated under an environment which has one change of stress levels, i.e., from severer stress level to milder one or from milder one to severer one. This method can be easily extended to the case where there is more than one change.

For modelling lifetime distributions under different stress levels, we will follow the ideas adopted in accelerated life tests (ALT). Accelerated life tests are frequently used in practice to obtain information on the life distribution or performance over time of highly reliable products in an affordable amount of testing time. During ALT, test units are used more frequently than usual or are subjected to higher level of stresses, such as temperature and voltage, than usual. The information obtained from the ALT is then used to predict product performance in the usual level of environment. Nelson (1990) provides an extensive and comprehensive source for background material, practical methodology, basic theory, and examples for accelerated testing. Meeker and Escobar (1993) proposed a general model to take care of the age conversion. Denote X as the lifetime of a component used in the usual level of environment and $F(t)$ as its Cdf. Let $f(t)$ and $r(t) = f(t)/\bar{F}(t)$ be the pdf and FR of X. Also let r.v. $X_{\mathcal{A}}$ be the lifetime of a component operated during ALT and $F_{\mathcal{A}}(t)$ be the corresponding Cdf. The 'Accelerated

Failure Time'(AFT) regression model is the most widely used parametric failure time regression model in ALT. Under this model higher stress has the effect of shrinking time through a scale factor. This can be expressed as

$$F_{\mathcal{A}}(t) = F(\rho \cdot t), \ \forall \ t \geq 0, \tag{1}$$

where ρ is a constant which depends on the accelerated stresses. As given in Section 3 of Meeker and Escobar (1993), a more general model can be expressed as

$$F_{\mathcal{A}}(t) = F(\rho(t)), \ \forall \ t \geq 0, \tag{2}$$

where $\rho(t)$ depends on the accelerated environment. Since the accelerated environment gives rise to higher stresses than usual environment, reasonable assumptions are $\rho \geq 1$ for the model (1) and $\rho(t) \geq t$ for all $t > 0$ and $\rho(0) = 0$ for the model (2). Furthermore we assume that $\rho(t)$ in the model (2) is strictly increasing, continuous and differentiable. Then, the models given in (1) and (2) imply that $X_{\mathcal{A}} \leq_{st} X$. Here, the notation "$\leq_{st}$" denotes the usual stochastic order, that is, we say that Z_1 is *smaller than* Z_2 *in the usual stochastic order*, denoted as $Z_1 \leq_{st} Z_2$ (or $F_1 \leq_{st} F_2$), if $F_2(t) \leq F_1(t)$, for all $t \geq 0$, where $F_1(t)$ and $F_2(t)$ are the Cdfs of Z_1 and Z_2, respectively.

Now we consider the case when the stress level changes from severer stress level to milder one at the system's operating time $t = b$. As before, denote X as the lifetime of a component used in the milder environment and $F(t)$ as its Cdf. Then the lifetime under severer environment can be modelled by (2). On the other hand, right after a new system has been operated during $(0, b]$ under the severer environment, the 'virtual age', which is transformed to the milder level of environment, of the system would be not less than b. Thus we assume that the survival function of the system with age b under severer environment, which is operated in the milder level of environment, is given by

$$\exp\left(-\int_0^t r(a(b)+u)du\right) = \frac{\bar{F}(a(b)+t)}{\bar{F}(a(b))} \equiv \bar{F}_b(t), \tag{3}$$

where $a(b)$ satisfies $a(b) \geq b$ for all $b \geq 0$, $a(0) = 0$ and is assumed to be strictly increasing and differentiable function. The equation (3) implies that the performance of a component with age b under severer environment is the same as that of a component which has been operated in the usual level of environment during time $(0, a(b)]$. Hence the function $a(b)$ represents the *accelerated ageing process* induced by the severer environment.

Now we consider the case when the stress level changes from milder stress level to severer one at the system's operating time $t = b$. Then, in this case, denote X as the lifetime of a component used in the severer environment and $F(t)$ as its Cdf. Then the lifetime under milder environment can also be modelled by (2) but with the modification of $\rho(t) \leq t$ for all $t > 0$. Then, the model given in (2) implies that $X \leq_{st} X_{\mathcal{A}}$. Also, by similar arguments discussed before, we assume that the survival function of the system with age b under milder environment, which is operated in the severer level of environment, is also given by (3) but with the modification of $a(b) \leq b$. Then, in this case, the function $a(b)$ represents the *delayed ageing process* induced by the milder environment.

3 Some Applications

In this section some applications of the method proposed in Section 2 will be considered.

3.1 Application to Accelerated Burn-in Procedure

In accelerated burn-in, a system is tested under severer environment and then it is operated under milder environment during field operation. Cha (2006) considered optimal accelerated burn-in time which optimizes various criteria. In accelerated burn-in, the stress level changes from severer stress level to milder one. Suppose that the accelerated burn-in time is b. Then from the model (2), the failure rate function in accelerated environment is given by

$$r_{\mathcal{A}}(t) = \frac{\rho'(t)f(\rho(t))}{1 - F(\rho(t))} = \rho'(t)r(\rho(t)),$$

where $\rho(t) \geq t$. From (3), it is easy to see that the burned-in component with accelerated burn-in time b and 'field use age' u has failure rate

$$r(a(b)+u),\ \forall\ u \geq 0,$$

where $a(b) \geq b$.

Now, combining the accelerated burn-in phase and the field use phase, the failure rate function of a component under accelerated burn-in time b, which is denoted by $\lambda_b(t)$, can be expressed as

$$\lambda_b(t) = \begin{cases} \rho'(t) r(\rho(t)), \text{ if } 0 \leq t \leq b \text{ (Burn-in Phase)}, \\ r(a(b)+(t-b)), \text{ if } t \geq b \text{ (Field Use Phase)}. \end{cases} \tag{4}$$

Cha (2006) developed several performance criteria based on the failure rate function given in (4), and studied optimal accelerated burn-in time.

3.2 Application to General Standby System

Cha et al. (2008) considered the reliability function of standby system when the lifetimes of the units in a standby system are any continuous r.v.s and study some related applications. Cha et al. (2008) considered a warm standby system with an active unit (unit 1) and a unit in warm standby (unit 2). The active unit is under surveillance by a switch, which activates the standby unit when the active unit fails. For simplicity of the model, it is assumed that the switchover to the standby unit is perfect, i.e. instantaneous and failure-free. In this case, the stress level of standby unit changes from milder stress level to severer one. Thus the modelling method discussed in Section 2 can be applied in modelling lifetime distribution of standby unit. Based on it, Cha et al. (2008) showed that the reliability function of the standby system is given by

$$\begin{aligned} R(t) &= \exp\{-\int_0^t \lambda(s)ds\} \\ &\quad + \int_0^t \exp\{-\int_0^{t-u} r(a(u)+s)ds\} \cdot \exp\{-\int_0^{\rho(u)} r(s)ds\} \cdot g(u)du, \end{aligned} \tag{5}$$

where $r(t)$ is the failure rate function of the warm standby unit (unit 2) under active state and $\lambda(t)$, $g(t)$ is the failure rate and probability density function of active unit (unit 1). Obviously, it should be assumed that $a(u) \leq u$ and $\gamma(u) \leq u$.

4 Concluding Remarks

In this paper, a general method for modelling lifetime distribution of a system which experiences one change of stress levels during its operation. The proposed method is based on the basic statistical property commonly used in accelerated life tests and the virtual age concept.

Since the proposed model is a general one, various specific parametric models for the functions $\rho(t)$ and $a(b)$ could be developed based on much experiences and extensive real data. Then, in the real applications, the proposed method would give the ways of general approach and the solutions for modelling lifetime distributions under different environments.

This method can be easily extended to the case where there is more than one change of stress levels in the operating environment.

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