On multicomponent system reliability with "weakly dependent" component life times

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Abstract

The goal of this presentation is to describe and define some, relatively new kind of **stochastic dependences** and their applications. The high generality of the defined dependences allows creating a large variety of stochastic models for, among others, **reliability** of complex (here, two component) parallel systems. By this we mean mostly new and analytically nice bivariate probability distributions, whose marginals are interpreted as the components' (dependent) life times.

At first, we give a short general description of a kind of system components' physical interactions. The resulting, "induced" by "physics of the problem" probabilistic relations between the random variables, in their joint probability distributions, we understand as stochastic **reflection of** rather complicated components' **physical interactions.** The interactions are assumed to obey some "continuous" pattern valid until one of the two components fails. According to that pattern a ("harmful") physical activity of one of the components produces, as a side effect an 'approximately continuous' string of 'micro-**shocks'.** As we assume, these micro-shocks affect the other component's failure mechanism, by causing a corresponding string of 'infinitesimal **micro-damages'** in its physical structure. Accumulation of these micro-damages in time can be considered in terms of the component's **degradation** or fatigue. In turn, the micro-damages are **related to** corresponding infinitesimal **increments** (or decrements) **in** that component's failure **rate.**

Based on this (hypothetical) association between the physical failure mechanism and its, much simpler probabilistic equivalence, we describe the latter analytically, in terms of the life-times' joint probability distribution.

Notice here, that the degree of 'one component (and so its life time) is physically affected by the other', is "proportional" to the (random) time the interaction lasted i.e., to that other component's life time, say X = x.

The considered stochastic "interactions between the components' life times" rely on an influence of one random variable (the X) on the other, indirectly **by influencing its probability distribution** rather than a particular realization y of the affected random variable Y. By that, we understand that the random variable, say X is transformed into original (corresponding to the defined "no interactions case") probability distribution G(y;) of the other component's life time, say Y.

In other words, instead of the usual "algebraic type" of a (direct) transformation 'X → Y'

we define its **"weak(er) (probabilistic) version":**

$\mathbf{X} \rightarrow \mathbf{G(y;} \ \theta \ \mathbf{)}.$

Realize that, unlike with the "strong (algebraic) version" of transformation, in the considered case a particular realization x, for an event (X = x), does **not** determine ('with probability one') the corresponding realization y of (Y = y). Instead, the **x** only **influences,** say the **probability** (density, if exists) **of the occurring** random event **(Y = y).** This, by the way, appears natural that a given life time x does not totally determine the other life time y, even if x still somehow ("weakly") influences the value y. More precisely, by the 'changes in the probability distribution we understand **changes in** the life times' hazard rates or, more generally, in the distributions' **parameter(s)** θ**.** Therefore the above weaker version $\mathbf{X} \rightarrow \mathbf{G(y;} \ \theta \ \mathbf{)}$ can be understood as a transformation of the random variable X into the parameter(s) θ.

The final version of the life times **transformation** can simply be expressed as: $\mathbf{X} \rightarrow \theta$.

From now, one can treat the "new parameter" θ of the Y's probability distribution as

a **function** of time x,for the event **X = x,** that also may be considered as "time of an endured stress". This allows employing the functional notation $\theta = \theta$ **(x).**

At the beginning stage, i.e., prior to eventual statistical testing of obtained data, the latter function is only assumed to be continuous. In such a way, we can define the conditional distribution (or, in the absolute continuous case, the conditional pdfs) of life time Y, given any event X = x, by the formula: $\mathbf{g_y(y|x) = g(y, \theta(x))}$,

where g(y ,) is the pdf corresponding to the, considered above, cdf G(y ,) of the Y.

As, according to our assumption the pdf f(x) of the life time X is given, we also obtain the **joint probability density h(x, y)** of the random vector **(X, Y),** as the simple arithmetic product:

$\mathbf{g(y,\ \theta(x))\ f(x).}$

Based on the same paradigm, extension to higher dimensions is straightforward. Actually, the described above method yields a tremendous number of new multivariate probability distributions. Many of them may be applied in a variety of the reliability problems.

The range of potential applications of the method probably largely exceeds the pure reliability framework.

Remark. The presented in this abstract new approach, or rather methodology, basically is absent in statistical literature. The one major exception, is the dependence structure of the classical multivariate normal, in particular, well seen in the expressions that define the conditional normal pdfs. Expressing them in terms of our theory, the originally constant expectation, say θ of any given normal density is "affected" by realizations of other normal variables to become the well known conditional expectation $\theta(x)$, where the latter function is linear. Our method, may essentially be seen as an extension of that multivariate normal distributions' paradigm. In this particular normal case the linear (regression) function can be replaced by any continuous one. The price of it is losing normality of one of marginals.