On Condition-based Maintenance Policy for Deteriorating System in Presence of Covariates

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Abstract

This paper discusses the problem of condition-based maintenance policy for a stochastic non-monotone deteriorating system in presence of covariates. The covariates process is assumed to be a finite-state Markov chain and the influence on deterioration is represented by a multiplicative exponential function. We derive the optimal maintenance threshold, optimal inspection sequence to minimise the expected average maintenance cost. Also we propose an adaptive maintenance policy to lower the maintenance cost.

1 Introduction

Condition-based maintenance policy is an important issue and it has been intensively studied in reliability engineering. In this paper, we determine optimum non-periodic inspection and maintenance policy for deteriorating systems with the influence of the covariates (also known as explanatory variables). The covariates process, which describes the dynamical environment under which the system operates ((Singpurwalla 1995), (Bagdonavičius and Nikulin 2000)), is assumed to be a temporally homogeneous Markov chain with finite state space. We model the degradation process by a stochastic univariate process, where the influence of the covariates is modelled by a multiplicative exponential function. We study the optimal non-periodic inspection/replacement policy for the considered non-monotone deteriorating system. The purpose of this paper is to propose an optimal maintenance policy for the considered system in order to minimise the global long-run expected average maintenance cost per time unit. We will also compare the global expected average maintenance cost for different cases of the covariates and different maintenance policies.

2 Stochastic deterioration process

Suppose that the system is subject to stochastic deterioration and the deterioration level can be inspected at discrete time units. Denote by D_n the level of the degradation at $n-$th time unit, and the system is supposed to have an initial degradation level $D_0 = 0$.

The covariates process $Z = \{Z_n, n \in \mathbb{N}^*\}$ is assumed to be a temporally homogenous Markov chain with finite state space $\{1, 2, \cdots, m\}$ describing the states of the environment. Let $p_{ij} = P(Z_{n+1} = j | Z_n = i)$, $\forall n \in \mathbb{N}^*$ be the transition probabilities of the process Z.

It is assumed that the degradation increment at time n depends only on the covariates at time n. One shall denote by $D_n(Z)$ the observed degradation level of the system at the $n-$th time unit under the influence of the covariates, defined by:

$$D_n(Z) = \max(D_{n-1}(Z) + X^+_{n-1}(Z_{n-1}) - X^-_{n-1}(Z_{n-1}), 0), \tag{1}$$

for $n \geq 1$, where $X^+_n(Z_n)$, $X^-_n(Z_n)$ are independent random variables (given Z_n), with exponential distributions with means $\mu^+(Z_n)$ and $\mu^-(Z_n)$ respectively. Without loss of generality, it is assumed that $\mu^+(Z_n) \geq \mu^-(Z_n)$ for a given Z_n.

To describe precisely the influence of covariates Z_n on $X^+_n(Z_n)$ and $X^-_n(Z_n)$, similarly to the proportional hazards model proposed by (Cox 1972), it is supposed that the parameters $\mu^+(Z_n)$ and $\mu^-(Z_n)$ depend on Z_n as follows:

$$\mu^+(Z_n) = \mu^+_0 \exp(\beta^+_{Z_n}), \quad \mu^-(Z_n) = \mu^-_0 \exp(\beta^-_{Z_n}), \tag{2}$$

where $\beta^+_{Z_n} = \langle \beta^+, \mathbf{I}_{Z_n} \rangle$ (resp. $\beta^-_{Z_n} = \langle \beta^-, \mathbf{I}_{Z_n} \rangle$) is the inner product of $\beta^+ = (\beta^+_1, \beta^+_2, \cdots, \beta^+_m) \in \mathbb{R}^m$ and $\mathbf{I}_{Z_n} = \left(\mathbf{1}_{(Z_n=1)}, \ldots, \mathbf{1}_{(Z_n=m)}\right)$ (the inner product of $\beta^- = (\beta^-_1, \beta^-_2, \cdots, \beta^-_m) \in \mathbb{R}^m$ and $\mathbf{I}_{Z_n}$), μ^+_0 and μ^-_0 are baseline degradation rate.

Let be $Z_0 = 1$ the covariates initial state and denote by π^n the distribution of Z_n defined as follows:

$$\pi^n = (\pi^n_1, \ldots, \pi^n_m) \text{ where } \pi^n_i = P(Z_n = i | Z_0 = 1) \text{ for } i = 1, \ldots, m.$$

We shall denote by $\pi = (\pi_1, \ldots, \pi_m)$ the stationary distribution of the Markov chain satisfying the following relation:

$$\lim_{n\to\infty} \pi^n_i = \pi_i,$$

3 Condition-based maintenance policy

Suppose that the system is a non-monotonically deteriorating stochastic system with initial state $D_0 = 0$, and the state can exclusively be monitored by inspection at non-periodic times $\Pi = \{T_1, T_2, \ldots, \}$. It is assumed that the inspections are perfect and all the maintenance actions are instantaneous. It is also assumed that the maintenance actions have no influence on the covariates process, that is, after each replacement, the covariates process $\{Z_n\}_{n\in\mathbb{N}^*}$ follows its trajectory.

3.1 Non Periodic inspection policy

The system is inspected at times $\Pi = \{T_1, T_2, \ldots\}$ where the inter-inspection intervals are defined sequentially and they depend on the deterioration level of the system. As the deterioration level of the system grows the inter-inspection interval decreases. After each inspection, the inter-inspection interval is defined by using an inter-inspection function depending on the deterioration level of the system ((Castanier, Bérenguer, and Grall 2003), (Newby and Barker 2009)). The sequence of inspections is defined by:

$$T_{n+1} = T_n + m(D_{T_n}), \tag{3}$$

where the inspection function $m(D_{T_n})$ is a decreasing function of the degradation level.

An example of a linear inter-inspection function is defined by ((Newby and Barker 2009)) as follows:

$$m(x) = \max\left\{1, a - \frac{a-1}{b}x\right\}, \tag{4}$$

where $a > 1$ corresponds to the length of the first inspection interval and $b \in \mathbb{R}^+$ controls the changes frequency of the inspection.

3.2 Replacement policy

We consider the case where two maintenance operations, preventive replacement and corrective replacement, are possible.

We shall denote by L the critical level of the degradation beyond which the system can not fulfil its mission correctly. If the deterioration level exceeds L the system is supposed to be failed. The corrective replacement takes place at the first inspection time after the failure:

$$G_L = \inf\{n \in \mathbb{N}^* : D_n \geq L | D_0 = 0\}.$$

Let T_p be defined by

$$T_p = \inf\{n \in \mathbb{N}^* : L_p \leq D_n \leq L | D_0 = 0\}. \tag{5}$$

Considering the non-monotonicity of the degradation process, the preventive replacement should take into account the recovering of the system. In difference with (Newby and Barker 2009), we don't consider process for which the last hitting time calculation is feasible. In order to take into account the possibility of recovering, the preventive replacement takes place if the degradation level stays within $[L_p, L)$ during R inspections following T_p defined by (5). Therefore the preventive replacement takes place at

$$H^R_{L_p} = R + 1 + \inf\{n \in \mathbb{N}^* : D_{T_n} < L_p \leq D_{T_{n+r+1}} < L, r \in \{0, 1, \ldots, R\}\}.$$

That is, suppose that $t = T_{n+1}$ is the first inspection satisfying $D_{T_{n+1}} \geq L_p$. The system will be maintained at $t = T_{n+R+1}$ (R inspections after the first exceeding time of the set $[0, L_p)$) if there are R successive observations within $[L_p, L)$.

3.3 Replacement policy with covariates

We will define and compare two different type of maintenance policies. These policies are defined as follows:

Global Maintenance: The maintenance decision rule doesn't take into account the state of the covariates. Let $L_p^*(Z)$, $a^*(Z)$ and $b^*(Z)$ be the parameters minimising the expected long-run average maintenance cost when the Z forms a general Markov chain. At each inspection time, we use $L_p = L_p^*(Z)$, $a = a^*(Z)$, $b = b^*(Z)$ as the maintenance decision rule.

Adaptive maintenance: At each inspection time, we take into account the state of covariates. Let $L_p^*(Z = i)$, $a^*(Z = i)$ and $b^*(Z = i)$ be the parameters minimising the expected long-run average maintenance cost when the process Z is static (the covariate is fixed to i at any time). In the adaptive decision rule, at each inspection time we use the inspection sequence parameters a, b and preventive threshold L_p according to the state of the covariates at that time, that is, if $Z_{T_k} = i$ at time T_k, the maintenance decision rule is based on $L_p = L_p^*(Z = i)$, $a = a^*(Z = i)$, $b = b^*(Z = i)$.

3.4 Evaluation of the maintenance policy

Each time that a maintenance action is performed on the system, a maintenance cost is incurred. Each corrective (respectively preventive) replacement entails a cost C_c (respectively C_p, $C_p < C_c$). The cost incurred at each inspection is C_i. In the period of unavailability of the system an additional cost per time unit C_d is incurred. The expected average cost is defined by:

$$EC_\infty(Z) = \lim_{t\to\infty} \frac{E(C(t,Z))}{t} = \lim_{t\to\infty} \frac{C_i E(N_i(t,Z)) + C_p E(N_p(t,Z)) + C_c E(N_c(t,Z)) + C_d E(d(t,Z))}{t}. \quad (6)$$

where $N_i(t,Z)$ (resp. $N_p(t,Z), N_c(t,Z)$) is the number of inspections (of preventive replacements, of corrective replacements) till time t and $d(t,Z)$ be the cumulative unavailability duration of the system before t.

The aim the optimisation is to find the optimal value of the threshold L_p^* and the parameters a^*, b^*, R^* such that

$$(L_p^*, a^*, b^*, R^*) = \arg \min_{(L_p,a,b,R)} EC_\infty(Z). \quad (7)$$

4 Numerical Simulation

In this section we give some numerical simulation of our model, the parameters are taken to be as follows: the transition matrix $P = \begin{pmatrix} 0.995 & 0.005 & 0.000 \\ 0.002 & 0.995 & 0.003 \\ 0.000 & 0.005 & 0.995 \end{pmatrix}$, the initial state $Z_0 = 1$, the regression parameters $\beta^+ = (0.2, 0.5, 1)$ and $\beta^- = (0.1, 0.1, 0.1)$, the baseline mean parameters $\mu_0^+ = 0.5$ and $\mu_0^- = 0.3$. The corrective threshold $L = 30$.

We consider four cases of unit maintenance cost: (1) Case I (Value of reference for costs): $C_i = 10, C_p = 60, C_F = 100, C_d = 250$; (2) Case II (Expensive preventive actions): $C_i = 10, C_p = 100, C_F = 100, C_d = 250$; (3) Case III (Expensive inspections): $C_i = 100, C_p = 60, C_F = 100, C_d = 250$, and (4) Case IV (cheap unavailability): $C_i = 10, C_p = 60, C_F = 100, C_d = 100$. We will also compare the following three quantities:

- optimal maintenance cost when Z forms a general Markov chain;
- optimal maintenance cost when Z_n are fixed to $Z = i$ $(i = 1, 2, 3)$ respectively;
- weighted mean of the optimum cost for $Z=i$ $(i=1,2,3)$ with the steady-state probability as weights:

$$EC_\infty = \sum_{k=1}^{3} EC_\infty^*(k)\pi_k. \quad (8)$$

- adaptive maintenance cost.

	Case I $(R^*, L_p^*, a^*, b^*, C^*)$	Case II $(R^*, L_p^*, a^*, b^*, C^*)$	Case III $(R^*, L_p^*, a^*, b^*, C^*)$	Case IV $(R^*, L_p^*, a^*, b^*, C^*)$
Z MC	(0, 18. 18, 22, 2.6519)	(0, 20, 18, 24, 3.6394)	(0, 8, 22, 24, 7.9278)	(0, 18, 20, 26, 2.4445)
$Z = 1$	(0, 22, 54, 24, 1.0884)	(0, 22, 52, 26, 1.5498)	(0, 8, 58, 26, 2.9289)	(0, 20, 54, 24, 1.0708)
$Z = 2$	(0, 20, 26, 26, 1.8807)	(0, 24, 30, 26, 2.5982)	(0, 14, 36, 26, 4.5880)	(0, 20, 32, 26, 1.7314)
$Z = 3$	(0, 18, 14, 24, 3.9043)	(0, 22, 16, 24, 5.3078)	(0, 10, 18, 24, 9.3525)	(0, 20, 14, 24, 3.7017)
$\bar{C}$	2.3293	3.2014	5.6855	2.1904
Adaptive	2.4758	3.3544	7.0843	2.3629

Table 1: The optimal inspection parameters and the optimal preventive threshold corresponding to the lowest maintenance cost

The optimisation parameters corresponding to the lowest maintenance cost with a linear inspection function (4) are exposed in Table 1.

It can be noticed in Table 1 that the weighted mean $\bar{C}$ is lower than the optimal cost corresponding to the case of covariates defined as Markov chain (MC). It is sensible that by decreasing the uncertainties (absence of Markov chain) one can decrease the maintenance cost.

In the framework of non periodic inspection policy the optimal delay ratio for R is $R^* = 0$. This value is obtained by the global minimisation of the maintenance cost as a function of four parameters R, L_p, a and b. The delay ratio is already taken into account in the non periodic inspection policy and there is no necessity in its presence.

The results of maintenance optimisation for different cases of unit maintenance cost shows that: the case II which corresponds to a costly preventive action results in more expensive than the normal case (case I); the case III which corresponds to a high unavailability cost and expensive inspections is the most expensive case, and the case IV which corresponds to a cheaper unavailability cost is the cheapest case and always close to the normal case (case I).

The maintenance costs corresponding to the adaptive policy are lower than maintenance costs of the global policy, which means that the use of the adaptive policy improves the quality of the maintenance policy. This is the case since in comparing to the other maintenance policies, we take into account an additional information upon the covariates process under the adaptive maintenance policy.

5 Conclusion

In this paper, considering the influence of the covariates, we study the non-periodic maintenance policy for a non-monotone deteriorating system. The covariates process is modelled by a finite state space Markov chain and its influence on system deterioration is evaluated by a multiplicative exponential function. In the framework of a non-monotone deterioration system, we find the optimal preventive threshold and optimal inspection sequence to minimise the maintenance cost per time unit. An adaptive maintenance policy, in which the information on covariates is taken into account, is proposed.

References

Bagdonavičius, V. and M. Nikulin (2000). Estimation in degradation models with explanatory variables. *Lifetime Data Analysis 7*, 85–103.

Castanier, B., C. Bérenguer, and A. Grall (2003). A sequential condition-based repair/replacement policy with non-periodic inspections for a system subject to continuous wear. *Applied Stochastic Models Business and Industry. 19*(4), 327–347.

Cox, D. (1972). Regression models and life-tables. *Journal of the Royal Statistical Society 34*, 187–220.

Newby, M. J. and C. T. Barker (2009). Optimal non-periodic inspection for multivariate degradation model. *"Reliability Engineering and System Safety" 94*, 33–43.

Singpurwalla, N. D. (1995). Survival in dynamic environnements. *Statistical Science 1*(10), 86–103.