Multi-state Markov Reward Models for Reliability Measures Calculation of Refrigeration System

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Abstract

This paper presents a method for calculation the reliability measures of a multi-state supermarket refrigeration system, where the system and its components can have different performance levels ranging from perfect functioning to complete failure. The suggested approach is based on the Markov reward models for computation of reliability measures for multi-state system. Corresponding procedure for reward matrix definition is suggested. A numerical example is presented in order to illustrate the approach.

1 Introduction

Supermarkets lose many millions of dollars each year due to stock losses from refrigerators in their stores. The most commonly used refrigeration system for supermarkets today is the multiplex (DX) direct expansion system. All display cases and cold store rooms use DX air-refrigerant coils that are connected to the system compressors in a remote machine room located in the back or on the roof of the store. Heat rejection is usually done with air-cooled condensers with axial blowers mounted outside. Due to the system's highly integrated nature, a fault in a single unit or item of machinery can't have detrimental effects on the entire store, only decrease of system cool capacity. Failure of compressor or axial condenser blower leads to partial system failure (degradation of output cooling capacity) as well as to complete failures of the system. We treat refrigeration system as multi-state system (MSS), where components and systems have an arbitrary finite number of states. According to the generic MSS model (Lisnianski and Levitin 2003), the system can have different states corresponding to the system's performance rates. The performance rate of the system at any instant $t \geq 0$ is a discrete-state continuous-time stochastic process.

In practice, the most commonly used MSS reliability measures are MSS availability, mean number of MSS failures during a fixed time interval [0, t], etc. In this paper, a generalized approach for the computation of main MSS reliability measures was suggested. The approach is based on the application of the Markov Reward Model. The MSS reliability measures can be found by corresponding rewards definitions for this model and then by using a standard procedure for finding an expected accumulated reward during the time interval [0,t] as a solution of the system of differential equations.

2 Model Description

The MSS behavior is characterized by its evolution in the space of states. The entire set of possible system states can be divided into two disjointed subsets corresponding to acceptable and unacceptable system functioning. MSS entrance into the subset of unacceptable states constitutes a failure. The system state acceptability depends on the relation between the MSS output performance and the desired level of this performance – demand that is determined outside of the system. In many practical cases, the MSS performance should be equal or exceed the demand.

The General Markov Reward Model considers the continuous time Markov chain with a set of states {1,…,k} and transition intensity matrix $\boldsymbol{a}=\left|a_{ij}\right|,\ i,j=1,...,k$. It is assumed that while the process is in any state i during any time unit, some money r_{ii} should be paid. It is also assumed that if there is a transition from state i to state j, the amount r_{ij} will be paid. The amounts r_{ii} and r_{ij} are called rewards. They can be negative while representing loss or penalty. The main problem is to find a total expected reward, accumulated up to time instant T under specific initial conditions. Let $V_i(t)$ be the total expected reward accumulated up to time t at state i. According to Howard (1960), the following system of differential equations must be solved under initial conditions in order to find the total expected reward:

$$\frac{dV_i(t)}{dt}=r_{ii}+\sum_{\substack{j=1\\ j\neq i}}^{k}a_{ij}r_{ij}+\sum_{j=1}^{k}a_{ij}V_j(t),\ i=1,...,k \tag{1}$$

MSS instantaneous (point) availability $A(t)$ is the probability that the MSS at instant $t>0$ is in one of the acceptable states. *The MSS average availability* $\bar{A}(T)$ is defined as mean fraction of time, when the system resides in the set of acceptable states during time interval [0,T]. In order to assess $\bar{A}(T)$ for MSS, the rewards in matrix r for MSS model should be determined by the following manner: (1) The rewards associated with all acceptable states should be defined as 1 and (2) The rewards associated with all unacceptable states should be zeroed as well as all rewards associated with transitions.

The mean reward $V_K(T)$ accumulated during interval [0, T] will define a part of time that MSS will be in the set of acceptable states in the case when state K is the initial state . This reward should be found as a solution of system (1). After solving (1) and finding $V_K(T)$, MSS instantaneous availability can be obtained as $\bar{A}(T)=V_K(T)/T$.

Mean number $N_f(T)$ of MSS failures during time interval [0, T]. This measure can be treated as a mean number of MSS entrances into the set of unacceptable states during time interval [0, T]. For its computation, the rewards associated with each transition from the set of acceptable states to the set of unacceptable states should be defined as 1. All other rewards should be zeroed. In this case, a mean accumulated reward $V_K(T)$ will define a mean number of entrances in an unacceptable area during a time interval [0, T]: $N_f(T)=V_K(T)$.

3 Numerical Example

Consider the refrigeration system used in one of the Israel supermarkets. The system consists of 4 compressors, situated in the machine room and 2 main and one reserved axial condenser blowers. The reserve blower begins to work only when one of the main blowers has failed. Compressor failure rate is one per year and 10 per year for the axial condenser blower. The mean repair time for the compressor is one month and for blower is 24 hours. The state-space diagram for the system is presented in Figures 2.

There are 19 states. In every circle is written the state number and system performance – cool capacity in BTU per year.

In states 1, 6, 11, 16 – all 4 compressors are on-line, in states 2, 7, 12, 17 – 3 compressors are on-line, in states 3, 8, 13, 18 – 2 compressors are on-line, 1n states 4, 9, 14, 19 – only one compressor is on-line, states 5, 10, 15 – failure of all 4 compressors. In states 1 – 5 two axial condenser blowers are on-line, in states 6-10 two blowers are on line (one main and one reserved), in states 11-15 only one blower is on line, in states 16 – 19 failure of all 3 blowers.

The required cool capacity demand is $5\bullet 10^9$ BTU per year, then there are 9 acceptable states – 1-3, 6-8 and 11-13. States 4, 5, 9, 10, 14, 15, 16-19 are unacceptable. The corresponding circles are filled grey.

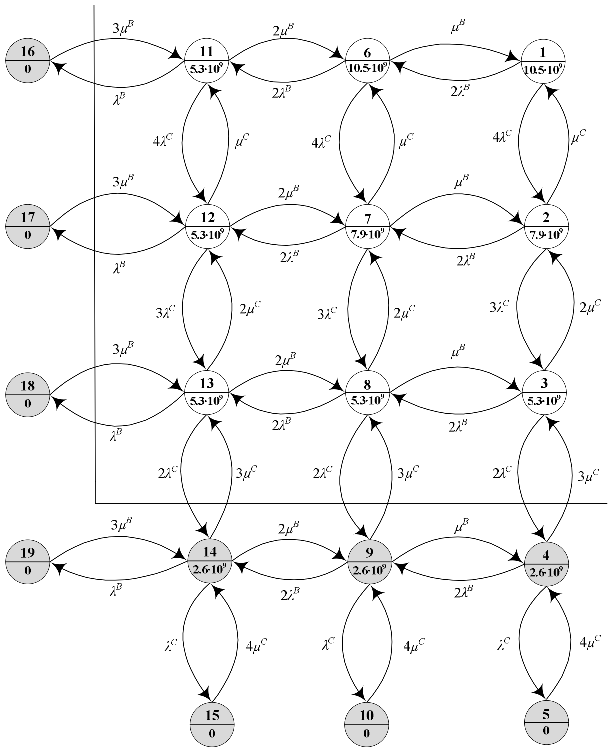

Figure 2: The state-space diagram for the refrigeration system

All transition intensities are shown in the Figure 2. The transition intensity matrix is shown below.

$$
\boldsymbol{a}=\begin{matrix}
\boldsymbol{a}_{11} & 4\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu^{C} & \boldsymbol{a}_{22} & 3\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2\mu^{C} & \boldsymbol{a}_{33} & 2\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3\mu^{C} & \boldsymbol{a}_{44} & \lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 4\mu^{C} & \boldsymbol{a}_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{66} & 4\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu^{B} & 0 & 0 & 0 & \mu^{C} & \boldsymbol{a}_{77} & 3\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu^{B} & 0 & 0 & 0 & 2\mu^{C} & \boldsymbol{a}_{88} & 2\lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \mu^{B} & 0 & 0 & 0 & 3\mu^{C} & \boldsymbol{a}_{99} & \lambda^{C} & 0 & 0 & 0 & 2\lambda^{B} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\mu^{C} & \boldsymbol{a}_{10,10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{11,11} & 4\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & \mu^{C} & \boldsymbol{a}_{12,12} & 3\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 2\mu^{C} & \boldsymbol{a}_{13,13} & 2\lambda^{C} & 0 & 0 & 0 & \lambda^{B} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\mu^{B} & 0 & 0 & 0 & 3\mu^{C} & \boldsymbol{a}_{14,14} & \lambda^{C} & 0 & 0 & 0 & \lambda^{B} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4\mu^{C} & \boldsymbol{a}_{15,15} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{16,16} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{17,17} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{18,18} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3\mu^{B} & 0 & 0 & 0 & 0 & \boldsymbol{a}_{19,19}
\end{matrix}
$$

where

$$a_{11}=\lambda_{12}+\lambda_{16} \qquad a_{77}=\lambda_{78}+\lambda_{7,12}+\mu_{72}+\mu_{76} \qquad a_{13,13}=\lambda_{13,14}+\lambda_{13,18}+\mu_{13,8}+\mu_{13,12}$$

$$a_{22}=\lambda_{23}+\lambda_{27}+\mu_{21} \qquad a_{88}=\lambda_{89}+\lambda_{8,13}+\mu_{83}+\mu_{87} \qquad a_{14,14}=\lambda_{14,15}+\lambda_{14,19}+\mu_{14,9}+\mu_{14,13}$$

$$a_{33}=\lambda_{34}+\lambda_{38}+\mu_{32} \qquad a_{99}=\lambda_{89}+\lambda_{8,13}+\mu_{94}+\mu_{98} \qquad a_{15,15}=\mu_{15,14}$$

$a_{44} = \lambda_{45} + \lambda_{49} + \mu_{43}$ $\quad a_{10,10} = \mu_{10,9}$ $\quad a_{16,16} = \mu_{16,11}$

$a_{55} = \mu_{54}$ $\quad a_{11,11} = \lambda_{11,12} + \lambda_{11,16} + \mu_{11,6}$ $\quad a_{17,17} = \mu_{17,12}$

$a_{66} = \lambda_{6,11} + \lambda_{67} + \mu_{61}$ $\quad a_{12,12} = \lambda_{12,13} + \lambda_{12,17} + \mu_{12,7} + \mu_{12,11}$ $\quad a_{18,18} = \mu_{18,13},\ a_{19,19} = \mu_{19,14}$

In order to find the MSS average availability $A(t)$ we should present the corresponding reward matrix in the following form:

$$\mathbf{r}^A = \left\{ r_{11} = r_{22} = r_{33} = r_{66} = r_{77} = r_{88} = r_{11,11} = r_{12,12} = r_{13,13} = 1, \text{ all other elements are zero} \right\}$$

In order to find the mean total number of system failures $N_f(t)$ we should present the corresponding reward matrix in the following form:

$$\mathbf{r}_{Nf} = \left\{ r_{34} = r_{89} = r_{13,14} = r_{11,16} = r_{12,17} = r_{13,18} = 1, \text{ all other elements are zero} \right\}$$

By solving the systems of differential equations (1) with transition intensity matrix $\boldsymbol{a}$ and reward matrixes $\mathbf{r}_A$ and $\mathbf{r}_{Nf}$ we can obtain an MSS average availability and mean total number of system failures during time period $[0, T]$, where $T = 5$ year. The results of calculation are presented in Figure 3. In addition here are presented results of calculation the same parameters for non-reserved system.

Curves in Figures 3 support the engineering decision-making and determine the areas where required reliability/availability level of the refrigeration system can be provided by configuration "with reserve" or by configuration "without reserve". For example, from the Figure 3A one can conclude that the configuration "without reserve" cannot provide the required average availability if it is greater than 0.988.

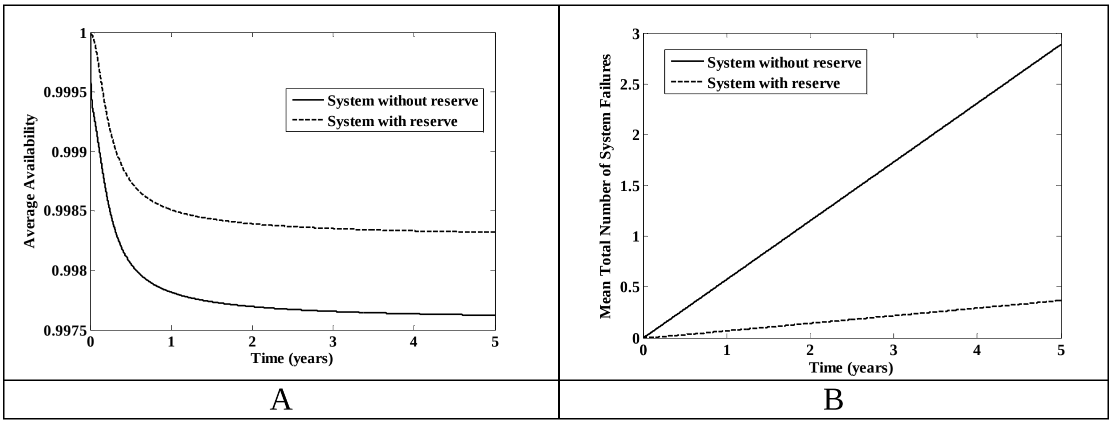

Figure 3: MSS average availability (A) and mean total number of system failures (B) for different types of systems

Conclusion

- The universal method was suggested to compute main MSS reliability measures. The method is based on different reward matrix determinations for an MSS model that is interpreted as a Markov Reward Model.
- The approach suggested is well formalized and suitable for practical application in reliability engineering. It supports the engineering decision-making and determines different system structures providing a required reliability/availability level of MSS.
- The numerical example is presented in order to illustrate the suggested approach.

References

[1] Howard, R. (1960). *Dynamic Programming and Markov Processes*. Cambridge, Massachusetts: MIT Press.

[2] Lisnianski, A. and Levitin, G. (2003). *Multi-state System Reliability. Assessment, Optimization and Applications*. NJ, London, Singapore: World Scientific.