Value of condition monitoring for a single-unit system subject to competing failures modes due to wear and traumatic events

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Abstract

This paper[1] develops the mathematical cost models for two maintenance policies (i.e. block replacement and periodic inspection/replacement) for a one-unit deteriorating system whose failures are due to the competing causes of accumulated wear and traumatic "shock" events. The value of the condition monitoring information obtained through the inspections is investigated by comparing on numerical examples the costs of both policies.

1 Introduction

We use in this work a failure model similar to a model proposed in [Singpurwalla 1995]. The system deterioration state is modeled by a stochastic process and the system is said to have failed whenever the accumulated wear exceeds a given failure level (i.e. deterioration-based failure) or when a traumatic event (e.g. shock) occurs, i.e. failures are due to the competing causes of accumulated wear and traumatic "shock" events. As advocated in [Singpurwalla 1995], such a failure/deterioration model can be seen as a combination and more versatile - and hopefully realistic - extension of more classical failure models based either only on deterioration or only on parametric lifetime distributions. The wear is described using a homogeneous Gamma process [van Noortwijk 2009] and the traumatic events arrive according to a non-homogeneous Poisson process with an intensity function which may also vary with the system wear level. We consider two maintenance policies for this system: a block replacement policy based on the knowledge of the lifetime distribution of the system, and an inspection/replacement policy in which the preventive replacement decision is made from the condition monitoring on the deterioration level returned by inspections. We develop the maintenance cost models associated to these policies and we use them to weigh the benefit of the condition information returned by the inspections against its cost.

2 System failure modeling

2.1 Deterioration-based failure

2.1.1 Deterioration modeling

We consider that the studied system deteriorates with usage and age. The accumulated deterioration (or wear level) at time t is modeled as a random aging variable $X(t)$. Without any repair or replacement action, $\{X(t), t \geq 0\}$ is a continuous-time monotonically increasing stochastic process, with $X(0) = 0$. We assume that the increments of wear are independent and that the deterioration process evolves in each time interval by means of an infinity of jumps. These characteristics lead to model the system wear as an homogeneous Gamma process, see [van Noortwijk 2009] for a thorough review on the use of Gamma processes in maintenance modelling: for all $0 \leq s \leq t$, the random wear increment $X(t) - X(s)$ has a gamma pdf with shape-parameter $\alpha \cdot (t-s)$ and scale parameter β:

$$f_{\alpha(t-s),\beta}(u) = \beta^{\alpha(t-s)} u^{\alpha(s-t)-1} e^{-\beta u} \cdot \mathbb{I}_{\{u \geq 0\}} / \Gamma(\alpha(t-s)) \tag{1}$$

where $\mathbb{I}$ and $\Gamma(\cdot)$ are the indicator function and Gamma function respectively. The average deterioration rate is $m = \alpha/\beta$ and its variance is $\sigma^2 = \alpha/\beta^2$. If the deterioration level $X(t)$ is greater than a fixed failure threshold L, the system is considered to be in the failed state due to a deterioration-based failure.

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2.1.2 On hitting times of the deterioration process

To characterize the deterioration-based failures, it is necessary to consider the hitting times of the Gamma process. Let σ_A be the time at which the deterioration reaches a level A, and F_{σ_A}, f_{σ_A} be respectively the distribution and density function of σ_A. Following the Gamma process deterioration model, we obtain:

$$F_{\sigma_A}(u) = \Gamma(\alpha u, \beta A)/\Gamma(\alpha u) \qquad \text{and} \qquad f_{\sigma_A}(u) = \alpha \int_0^A f_{au,\beta}(s) \cdot (\psi(\alpha u) - \ln(\beta s))\, ds$$

where $\Gamma(\alpha u, \beta A) = \int_{\beta A}^{\infty} z^{\alpha u - 1} e^{-z} dz$ (i.e. the incomplete Gamma function) and $\psi(v) = \Gamma'(v)/\Gamma(v)$ (i.e. the Digamma function). Considering another level B ($B > A$), we will see below (e.g. in Eq. 5) that the survival function of $\sigma_B - \sigma_A$ is also of interest but much more difficult to derive because of "overshoot behavior" of Gamma processes, i.e. $X(\sigma_A) \neq A$ and $\sigma_B - \sigma_A \neq \sigma_{B-A}$. The exact survival function of $\sigma_B - \sigma_A$ has been derived in [Bérenguer, Dieulle, Grall, and Roussignol 2003], but we use in this work the following approximation, much easier to compute and to handle:

$$F_{\sigma_B - \sigma_A}(u) \simeq F_{\sigma_{B-A-\frac{1}{2\beta}}}(u) \qquad \text{and} \qquad f_{\sigma_B - \sigma_A}(u) \simeq f_{\sigma_{B-A-\frac{1}{2\beta}}}(u)$$

2.2 Internal failure or traumatic event modeling

The system is also subject to traumatic events which cause its failure, i.e. "internal failure". These traumatic events arrive according to a non-homogeneous Poisson process whose intensity depends on the system deterioration level through the following relation:

$$r(t) = r(X(t)) = \begin{cases} \lambda_1 & X(t) \leq M_s \\ \lambda_2 & M_s < X(t) < L \end{cases} \tag{2}$$

where M_s is a predetermined wear threshold. Generally, $\lambda_2 > \lambda_1$ which means that the system is more prone to these internal failures when the wear level increases and exceeds a value M_s.

3 Maintenance policies

Two maintenance policies are considered in this section: a block replacement policy and a periodic inspection and replacement policy. The system starts its operation at time $t = 0$ and a replacement (either preventive or corrective) corresponds to a perfect and instantaneous renewal and restores the system to an as-good-as-new condition. Because of these renewal points, and using the renewal theorem, the long run expected maintenance cost rate of the maintained system can be computed on a single renewal cycle (i.e. between two successive AGAN replacements) as follows:

$$EC_\infty = \lim_{t \to +\infty} \{\mathbb{E}[C(t)]/t\} = \mathbb{E}[C(T_c)]/\mathbb{E}[T_c] \tag{3}$$

where T_c is length of a renewal cycle and $C(\cdot)$ is the cumulative maintenance cost.

3.1 Block replacement policy

3.1.1 Block replacement policy structure

Under the considered block replacement policy, the system is replaced at regular time intervals T, either preventively if it still running at the end of the replacement interval, or correctively if a failure occurred since the last replacement. A preventive replacement is performed with a cost C_p. When the system fails (due to either wear-based or internal "shock' failures) before the end of the replacement period, it remains failed until the scheduled replacement time. Thus, a failure generates both a corrective replacement cost $C_c > C_p$ and cost for the inactivity of the system at a cost rate C_d. The replacement period T is the only decision variable for this policy.

3.1.2 Cost model

The cost criterion for this classical policy is given by

$$EC_\infty(T) = \left(C_p \cdot \overline{F}_f(T) + C_c \cdot \left(1 - \overline{F}_f(T)\right) + C_d \cdot \int_0^T F_f(t) \cdot dt \right) / T \tag{4}$$

where $\overline{F}_f(t)$ is the system survival function with respect to both deterioration-based and internal failure. $\overline{F}_f(t) = \mathbb{P}\left(X\left(t\right) \leq L, N(t) = 0\right)$ where $N\left(t\right)$ is the number of internal failures in interval $[0, t]$and it can be shown that

$$\overline{F}_f(t) = \overline{F}_1(t) \cdot \overline{F}_{\sigma_{M_s}}(t) + \int_{u=0}^{t} \overline{F}_{\sigma_L - \sigma_{M_s}}\left(t-u\right) \cdot \frac{\overline{F}_1(u)\overline{F}_2(t)}{\overline{F}_2(u)} \cdot f_{\sigma_{M_s}}\left(u\right) \cdot du \tag{5}$$

where σ_{M_s} is the hitting time of the level M_s by the deterioration process, $\overline{F}_{\sigma_{M_s}}$ is the associated survival function and $\overline{F}_1$ (resp. $\overline{F}_2$) is the unconditional survival function with respect to internal failures with the failure rate λ_1 (resp. λ_2). The optimal replacement time T^* is obtained by the expression $C\left(T^*\right) = \inf_{T} \left\{EC_\infty\left(T\right), T > 0\right\}$

3.2 Periodic inspection and replacement policy

We consider now a more sophisticated maintenance policy in the sense that the maintenance decision is based on the condition monitoring information returned by inspections on the system.

3.2.1 Policy structure

The system is periodically inspected with period T. An inspection gives information on the system state by returning its deterioration level, but incurs a cost C_i. If the observed wear level of the system exceeds a threshold M and if no failure occurred the system is preventively replaced with a cost $C_p > C_i$. When upon inspection the system is detected in the failed state (due to wear or internal failures), the system is correctively replaced with a cost $C_c > C_p$. In this case, because of the system inactivity after failure, an additional cost is incurred from the failure time until the inspection time at a cost rate C_d. The two decision variables of this policy are the inter-inspection time T and the preventive threshold M.

3.2.2 Cost model

It can be shown [Huynh, Barros, Bérenguer, and Castro 2009] that the expected cost rate for the periodic inspection and replacement policy, computed on a renewal cycle, is given by

$$EC_\infty(T, M) = \frac{C_p \cdot \mathbb{P}_p(T, M) + C_c \cdot (1 - \mathbb{P}_p(T, M)) + C_d \cdot \mathbb{E}\left[W_r\right]}{\mathbb{E}\left[T_c\right]} + C_i/T \tag{6}$$

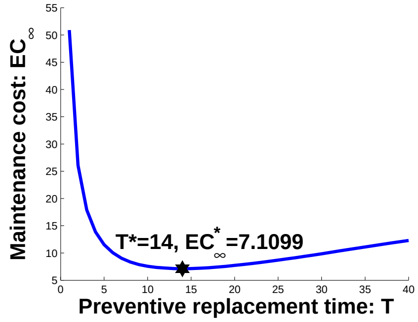
where $\mathbb{E}\left[T_c\right] = \sum_{k=0}^{\infty} \left(k+1\right)T \cdot \mathbb{P}\left(\text{Replacement at } \left(k+1\right)T\right)$ is the expected length of a renewal cycle, $\mathbb{P}_p(T, M) = \sum_{k=0}^{\infty} \mathbb{P}\left(kT < \sigma_M < \left(k+1\right)T < \sigma_L, N\left(\left(k+1\right)T\right) = 0\right)$ is the probability of a preventive replacement, $\mathbb{E}\left[W_r\right] = \sum_{k=0}^{\infty} \int_{kT}^{(k+1)T} \mathbb{P}_{d,k}(t)dt$ is the expected downtime in a cycle and $\mathbb{P}_{d,k}(t)$ denotes the probability of inactivity of the system at t ($kT \leq t \leq (k+1)T$).

The optimal values of the decision variables T^* and M^* are such that $EC_\infty\left(T^*, M^*\right) = \inf_{T,M} \left\{EC_\infty\left(T, M\right), T > 0, 0 < M < L\right\}$.

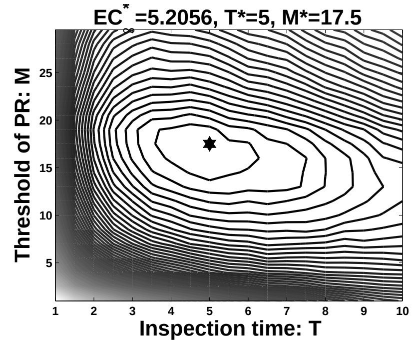
4 Numerical results and discussion

Figure 1 shows the expected maintenance cost for both policies as a function of the decision variables and illustrate in both cases the existence of an optimal tuning of the policy parameters.

In order to illustrate a possible use of our model to assess the value of the condition monitoring information, we varied the inspection cost C_i and investigated the corresponding evolution of the total maintenance of the inspection/replacement policy. The comparison of this cost to the cost incurred by the block replacement policy allows to weigh the benefit of the condition information returned by the inspections against its cost. Figure 2 presents this comparison for different deterioration/failure behaviors corresponding to different variances of the Gamma process and different intensities for the traumatic events process. These results show clearly that for deterioration processes with high variances in the increments, the inspection/replacement policy may lead to substantial savings in the maintenance costs. In these cases, it is indeed useful to follow closely the actual evolution of the deterioration path to adapt the maintenance decisions to the true state of the system, instead of applying the "static" rule of the block replacement policy based only on the "a priori" system lifetime distribution. The analysis of these costs savings could be used to justify or not the choice to implement an inspection/replacement policy based on condition monitoring and to invest in condition monitoring devices.

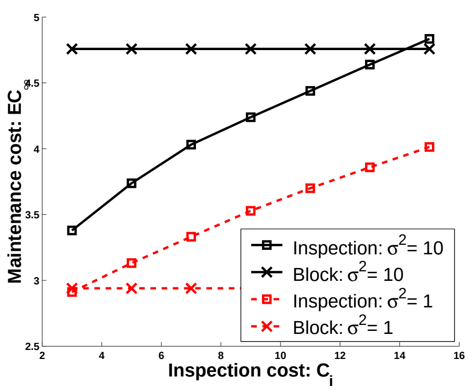


(a) Block replacement policy - $T^* = 14$, $EC^*_\infty = 7.11$

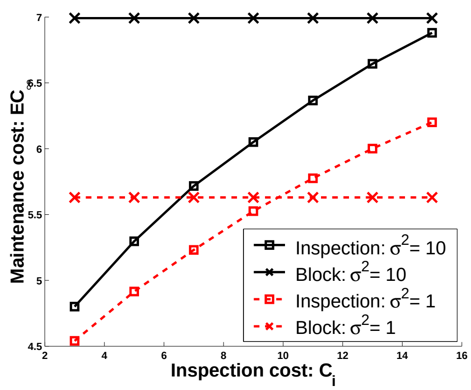


(b) Iso-level cost curves for inspection/replacement policy - $C_i = 2$, $Ms = 20$, $T^* = 5$, $M^* = 17.5$, $EC^*_\infty = 5.21$

Figure 1: Optimal values of maintenance policies for $\alpha = \beta = 0.1$, $L = 30$, $M_s = 20$, $\lambda_1 = 0.01$, $\lambda_2 = 0.1$, $C_p = 50$, $C_c = 100$, $C_d = 25$



(a) $\lambda_1 = 0.001$, $\lambda_2 = 0.01$



(b) $\lambda_1 = 0.01$, $\lambda_2 = 0.1$

Figure 2: Comparison of the policies maintenance costs as a function of the inspection cost C_i - $L = 30$, $M_s = 20$, $C_p = 50$, $C_c = 100$, $C_d = 25$

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