Open heterogeneous queueing networks with multiregime service strategies

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Abstract

The model of open queueing network with multiregime service strategies, several types of positive and negative customers is considered. Besides, there are signals, which can increase or reduce the regime of service in the node. The problem of stationary distribution form is investigated.

1 Introduction

Stationary distribution underlies calculations of basic features of network work, which is necessary at engineering and operating of real network objects. A great number of models, which allow to consider the request of modernity, have been recently appeared. Networks with multiregime service strategies allow to consider the models, in which service canals are partly unreliable and can work in different regimes. It depends on the degree of reliability of service canal. In this case we can consider partial loss of efficiency. Hence, we have the reduction of service rate.

2 Model description

We consider open queueing network with M types of positive and negative customers, which contains N nodes. There are four Poisson input flows: the flow of positive customers with parameter λ^+, the flow of negative customers with parameter λ^- and two flows of signals, which can reduce or increase the number of regime. They have rates ω^- and ω^+ accordingly.
Incoming positive customer of any type increases the length of queue and needs service. Negative customer doesn't need service, it "kills"positive customer of corresponding type and disappears. Negative customer doesn't produce any action, if the node doesn't have any positive customer of corresponding type.
Every node has the only service canal, which can work in $r_i + 1, i = \overline{1,N}$, regimes. We define 0 as the basic regime. Switching time from some regime to another one has the exponential distribution. While the regimes are switching in the node, the number of customers doesn't change. Switch occurs only between the neighborhood regimes.
When the signal of regime reducing incomes to the node with the regime l, it turns the node to the regime $l-1$ and doesn't change the number of customers in the node. This signal doesn't produce any action, if the node is in the regime 0. When the signal of regime increasing incomes to the node with the regime l, it turns the node to the regime $l+1$ and doesn't change the number of customers in the node. This signal doesn't produce any action, if the node is in the regime r. After changing the node regime these signals disappear.
Service times of customers are independent and have exponential distribution with parameter $\mu_{i,u}$ for positive customers of type u, $u = \overline{1,M}$ in the node i, $i = \overline{1,N}$.
Every customer of input flow of positive customers passes independently to node i and becomes the customer of type u with probability $p^+_{0(i,u)}$ $\left(\sum_{i=1}^{N}\sum_{u=1}^{M} p^+_{o(i,u)} = 1\right)$. Every customer of input flow of negative customers passes independently to node i and becomes the customer of type u with probability $p^-_{0(i,u)}$ $\left(\sum_{i=1}^{N}\sum_{u=1}^{M} p^-_{o(i,u)} = 1\right)$. Incoming signal of regime increasing and signal of regime reducing pass to node i with probabilities q^+_{0i} and q^-_{0i} accordingly $\left(\sum_{i=1}^{N} q^+_{oi} = 1, \sum_{i=1}^{N} q^-_{oi} = 1\right)$. After the service in node i positive customer of type u passes to node j immediately with probability $p^+_{(i,u)(j,v)}$ as positive of type v,with probability $p^-_{(i,u)(j,v)}$ as negative of type v and as the signal of increasing or reducing

regime with probabilities $q^+_{(i,u)j}$, $q^-_{(i,u)j}$ accordingly. Or it can leave the node with probability $p_{(i,u)0}$ $\left(\sum_{j=1}^N \sum_{v=1}^M (p^+_{(i,u)(j,v)} + p^-_{(i,u)(j,v)} + q^+_{(i,u)j} + q^-_{(i,u)j}) + p_{(i,u)0} = 1\right)$.

The state of the network is characterized by the vector $x(t) = (x_1(t), ..., x_N(t))$, where $x_i(t)$ describes the state of node i at the moment t. Denote $|x_i|^{l_i}_u$ as the quantity of positive customer of type u , $u = \overline{1,M}$, in node i, $i = \overline{1,N}$,which is working at l_i regime, when the network is in the state x_i.

When positive customer of type u comes to node i from the outside or from the other node, it removes the node from the state x_i to the state $\tilde{x}_i$ with probability $\pi^+_{i,u}(x_i, \tilde{x}_i)$ $\left(\sum_{u=1}^M \sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u + 1} \pi^+_{i,u}(x_i, \tilde{x}_i) = 1\right)$; negative customer - with probability $\pi^-_{i,u}(x_i, \tilde{x}_i)$ $\left(\sum_{u=1}^M \sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u - 1} \pi^-_{i,u}(x_i, \tilde{x}_i) = 1\right)$; signal, which increases regime, – with probability $\sigma^+_i(x_i, \tilde{x}_i)$ $\left(\sum_{u=1}^M \sum_{|\tilde{x}_i|^{l_i+1}_u = |x_i|^{l_i}_u, \tilde{x}_i \neq x_i} \sigma^+_i(x_i, \tilde{x}_i) = 1\right)$; signal, which reduces regime, – with probability $\sigma^-_i(x_i, \tilde{x}_i)$ $\left(\sum_{u=1}^M \sum_{|\tilde{x}_i|^{l_i-1}_u = |x_i|^{l_i}_u, \tilde{x}_i \neq x_i} \sigma^-_i(x_i, \tilde{x}_i) = 1\right)$.

Positive customer, which has received its service and leaves node i, removes the node from the state x_i to the state $\tilde{x}_i$ with probability $\rho_{i,u}(x_i, \tilde{x}_i)$, $\left(\sum_{u=1}^M \sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u - 1} \rho_{i,u}(x_i, \tilde{x}_i) = 1\right)$.

It is supposed that internal transitions from the state x_i to another state $\tilde{x}_i$ are possible in every node of the network. Herewith the number of positive customer in the node keeps the same ($|\tilde{x}_i|^{l_i+1}_u = |x_i|^{l_i}_u, |\tilde{x}_i|^{l_i-1}_u = |x_i|^{l_i}_u, \tilde{x}_i \neq x_i$) . It means that the transition from x_i to $\tilde{x}_i$ doesn't bound up with arrivals or departures of customers or signals. But it bounds up with transitions of the node from one regime to another. For the states, which have the number of regime $1 \leq l \leq r - 1$, the time of residence in the regime l has an exponential distribution. The node passes to $l + 1$ regime with rate $\nu_i(x_i, \tilde{x}_i)$ and to $l - 1$ regime with rate $\varphi_i(x_i, \tilde{x}_i)$. It is supposed that $\nu_i(x_i, \tilde{x}_i) = 0$, when the node is in the regime r and $\varphi_i(x_i, \tilde{x}_i) = 0$, when the node is in the regime 0.

3 Isolated node

We consider isolated node i and suppose that four independent Poisson flows come in it: the flow of positive customers of type u with parameter $\alpha^+_{i,u}$, the flow of negative customers of type u with parameter $\alpha^-_{i,u}$, the flow of signals, which increase regime of the node, with parameter β^+_i and the flow of signals, which reduce regime of the node, with parameter β^-_i. Here $\alpha^+_{i,u}$, $\alpha^-_{i,u}$, β^+_i, β^-_i – average rates of positive, negative customers, "increasing"signals and "reducing"signals arrivals accordingly to the node i.

Denote $p_i(x_i)$ as final stationary probabilities of states of Markov process $x(t)$, which describes the network model. The reversibility equations for isolated node have the next form:

$$\alpha^+_{i,u}\pi^+_{i,u}(x_i, \tilde{x}_i)p_i(x_i) = [\mu_{i,u}\rho_{i,u}(\tilde{x}_i, x_i) + \alpha^-_{i,u}\pi^-_{i,u}(\tilde{x}_i, x_i)]p_i(\tilde{x}_i), |\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u + 1, u = \overline{1,M}; \quad (1)$$

$$[\beta^+_i\sigma^+_i(x_i, \tilde{x}_i) + \nu_i(x_i, \tilde{x}_i)]p_i(x_i) = [\beta^-_i\sigma^-_i(\tilde{x}_i, x_i) + \varphi_i(\tilde{x}_i, x_i)]p_i(\tilde{x}_i), |\tilde{x}_i|^{l_i+1}_u = |x_i|^{l_i}_u, \tilde{x}_i \neq x_i, u = \overline{1,M}. \quad (2)$$

Traffic equations for this model are:

$$\alpha^+_{i,u} = \lambda^+ p^+_{0(i,u)} + \sum_{j=1}^N \sum_{v=1}^M \frac{\alpha^+_{j,v}\mu_{j,v}}{\alpha^-_{j,v} + \mu_{j,v}} p^+_{(j,v)(i,u)},$$

$$\alpha^-_{i,u} = \lambda^- p^-_{0(i,u)} + \sum_{j=1}^N \sum_{v=1}^M \frac{\alpha^+_{j,v}\mu_{j,v}}{\alpha^-_{j,v} + \mu_{j,v}} p^-_{(j,v)(i,u)},$$

$$\beta^+_i = \omega^+ q^+_{0i} + \sum_{j=1}^N \sum_{v=1}^M \frac{\alpha^+_{j,v}\mu_{j,v}}{\alpha^-_{j,v} + \mu_{j,v}} q^+_{(j,v)i},$$

$$\beta^-_i = \omega^- q^-_{0i} + \sum_{j=1}^N \sum_{v=1}^M \frac{\alpha^+_{j,v}\mu_{j,v}}{\alpha^-_{j,v} + \mu_{j,v}} q^-_{(j,v)i}.$$

We suppose, that the next condition holds

$$\sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u + 1} p_i(\tilde{x}_i)\rho_{i,u}(\tilde{x}_i, x_i) = \sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u + 1} p_i(\tilde{x}_i)\pi^-_{i,u}(\tilde{x}_i, x_i), i = \overline{1,N}, u = \overline{1,M}, l_i = \overline{0,r_i}. \quad (3)$$

4 Main result

If inequalities

$$\alpha^+_{i,u} < \mu_{i,u} + \alpha^-_{i,u}, \qquad \sup_{x_i} \sum_{|\tilde{x}_i|^{l_i}_u = |x_i|^{l_i}_u, \tilde{x}_i \neq x_i} (\nu_i(x_i, \tilde{x}_i) + \varphi_i(x_i, \tilde{x}_i)) < \infty$$

and condition (3) hold for all i, $i = \overline{1, N}$, and u, $u = \overline{1, M}$, then Markov process is ergodic and its stationary distribution has product form

$$p(x) = p_1(x_1)p_2(x_2)...p_N(x_N),$$

where $p_i(x_i)$ – stationary distribution of isolated node. We can determine it from (1) and (2). The method of reverse time was applied for the proof of this theorem.

Conclusion

We consider open queueing network with several types of positive and negative customers, two types of signals and multiregime service strategies. Incoming flows are Poisson. Service times of customers are independent and have exponential distribution. Single server in the node can operate in several regimes. Switch occurs only between the neighborhood regimes. The stationary network state distribution are determinated in product form.

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