Exponential Asymptotics for Nonlinearly Perturbed Renewal Equation with Non-polynomial Perturbations

Ying Ni
Mälardalen University
721 23 Västerås
Sweden
ying.ni@mdh.se

Abstract

This communication presents results from paper (Ni, Silvestrov and Malyarenko 2008), where a special type of nonlinearly perturbed continuous-time renewal equation is studied. Models of nonlinearly perturbed renewal equations have been actively studied in the last decades. The recent book (Gyllenberg and Silvestrov 2008) can be referred for the theory of nonlinearly perturbed renewal equations and their applications to analysis of nonlinearly perturbed stochastic processes and systems. The novelty of our results is that a new type of non-polynomial perturbations is concerned, It involves asymptotic expansions with respect to asymptotic scale $\{\varphi_{n,m}(\varepsilon) = \varepsilon^{n+m\omega}\}$, with n, m being non-negative integers and $\omega > 1$ being some irrational number. More specifically, the perturbation conditions mean that the defect and the moments of distribution F_ε generating the renewal equation are nonlinear functions of the perturbation parameter and can be expanded with respect to the asymptotic scale pointed above. The main result of the paper is a theorem that presents exponential asymptotic expansions for solutions of perturbed renewal equations with the aforementioned non-polynomial type of perturbations. This result may have many potential applications, for instances, to analysis of quasi-stationary phenomena for nonlinearly perturbed queuing systems, reliability models and risk processes. In paper (Ni, Silvestrov and Malyarenko 2008), risk processes with perturbations described above are considered and asymptotic expansions are given for ruin probabilities in the diffusion approximation model.

References

[1] Gyllenberg, M., Silvestrov, D.S. (2008). *Quasi-stationary Phenomena in Nonlinearly Perturbed Stochastic Systems*. De Gruyter Expositions in Mathematics, **44**, Berlin: Walter de Gruyter.

[2] Ni,Y., Silvestrov, D. and Malyarenko, A. (2008). Exponential Asymptotics for Nonlinearly Perturbed Renewal Equation with Non-polynomial Perturbations. *J. Numer. Appl. Math.*, **1(96)**, 173-197.