Reliability Prediction of an Imperfect Multi-stage Production System with Spares subject to Weibull Failures

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Abstract

Reliability prediction plays a very important role in system design and evaluation. In order to accurately predict the system reliability, one should consider the system configuration and the failure distribution of its components. This paper discusses a multi-stage production system with one machine/tool in active state and n spares in standby state. When the operating machine/tool breaks down, a switching device detects the machine failure via the sensor and the defective tool is replaced with a functional spare, so the system can resume its operation.

The Weibull Distribution is one of the most flexible failure distributions which is widely used because it can adequately describe the reliability behavior during the lifetime of present day systems. This paper assumes the operating machines/tools follow Weibull Failures, but the spares, sensor and switch failures follow exponential distribution. In addition, three assumptions are made in regard to its switch failures: (1) under the energized condition (2) under the failing-open condition (3) under the failing-closed condition.

Due to the intractability of Weibull Distribtution in such a imperfect switching system, it is difficult to solve the multiple integration involved analytically. Therefore, a recursive algorithm using numerical integration method is developed for predicting the reliability of a multi-stage production system with m imperfect switching subsystems subject to Weibull Failures. Finally, a numerical example is also given to explain and demonstrate the practical application of the developed reliability prediction model.

1 INTRODUCTION AND LITERATURE REVIEW

Considerable amount of research has been done in the area of modelling standby systems with various assumptions about the failure distribution, repair time distribution and switching failure mechanism etc. Table 1 provides a detailed summary of various assumptions considered in predicting the reliability of standby systems by various authors. This paper is an extension of the research efforts of Pan and Sun (1995) and Pan (1997). Considering the time-to-failure distributions of components follow Weibull failures, the development of mathematical models for predicting various imperfect switching/ multi-stage production systems is outlined. A manufacturing example which demonstrates the practical application of the developed reliability prediction model is also illustrated in this paper.

2 DEVELOPMENT OF RELIABILITY MODELS

2.1 The mathematical notations and assumptions of imperfect switching system

The system configuration, as depicted by Figure 1, consists of n+1 identical components, component 1 is in active state, the rest of n components is in standby state. The time-to-failure distribution of component 1 (in active state) follows Weibull probability distribution,

$$f(t) \quad = \quad \alpha\beta t^{\beta-1}\exp\left[\alpha t^{\beta}\right], t \geq 0;\ \alpha,\beta > 0,$$

where α is a scale parameter and β is a shape parameter.This system also has one switch and one sensor which are subject to failure. The mathematical notations used in this paper are defined as follows. λ_q: failure rate of a component when failure occurs in standby state; λ_{swe}: failure rate of the switch in energized model; λ_{swo}: failure rate of the switch in failing open mode; λ_{swc}: failure rate of the switch in failing closed mode; λ_{se}: failure rate of the sensing unit; $mstage$: the number of stages in a series system; CRs: the current system reliability; n:the number of components for each stage; Rs: the stage (subsystem) reliability.

Table 1: Pertinent literature on reliability modelling of standby systems with various assumptions.

Authors	System configuration	Failure distribution	Failure mechanism of the switch *	Switching type	Repair distribution	Constraints	Methodology
Chow (1971)	2-unit redundant system	Exponential	Random	Imperfect	N/A	N/A	Probabilistic method
Nakagawa (1977)	2-unit redundant system	General	swc swo	Imperfect	N/A	N/A	Markov renewal theory
Pan and Kolarik (1984)	Series system with spares	Weibull	---	Perfect	N/A	Cost	Numerical method
Kececioglu (1990)	n-component redundant system	Exponential	cyclic swo swc	Imperfect	Exponential	N/A	Makov chain Laplace transformation
Pan and Sun (1995)	n-component redundant system	Gamma	swe swo swc	Imperfect	Gamma	N/A	Makov chain Laplace transformation
Pan (Proposed in this paper)	Series system with spares	Weibull	swe swo swc	Imperfect	N/A	Cost	Numerical method

* Note: swc indicates failing-closed for the switch.
swo indicates failing-open for the switch.
swe indicates failing-energized for the switch.

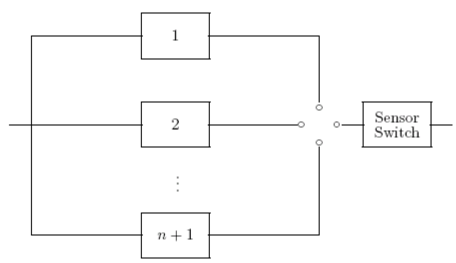

Figure 1: System configuration with n standby components, one switch and one sensor

2.2 The system reliability of a three-component imperfect switching system

The success modes of a three component system are shown in Figure 2. The four success modes are dercribed below:

The probability of success Model I can be written as:

$$R_{\mathrm{I}}(t) = e^{-\alpha t^{\beta}-\lambda_{swo}t} \quad (1)$$

The probability of success Model II can be written as:

$$R_{\mathrm{II}}(t) = \alpha\beta e^{-\lambda_{swe}-\lambda_{swo}t}\int_0^t t_1^{\beta-1}e^{-(\lambda_{swc}-\lambda_{se}-\lambda_q)t_1-\alpha t_1^{\beta}-\alpha(t-t_1)^{\beta}}dt_1 \quad (2)$$

The probability of success Model III can be written as:

$$R_{\mathrm{III}}(t) = \alpha\beta\lambda_q e^{-2\lambda_{swe}-\lambda_{swo}t}\int_0^t t_1^{\beta-1}e^{-\alpha t_1^{\beta}-\lambda_{se}t_1-\lambda_{swc}t_1-2\lambda_q t_1-\alpha(t-t_1)^{\beta}}dt_1 \quad (3)$$

The probability of success Model IV can be written as:

$$R_{\mathrm{IV}}(t) = (\alpha\beta)^2 e^{-\lambda_{swo}-2\lambda_{swe}}\left[\int_0^t t_1^{\beta-1}e^{-\alpha t_1^{\beta}-(\lambda_{swc}+\lambda_{se}+2\lambda_q)t_1} \times \int_0^{t-t_1} t_2^{\beta-1}e^{-\alpha t_2^{\beta}-(\lambda_{swc}+\lambda_{se}+\lambda_q)t_2-\alpha(t-t_1-t_2)^{\beta}}dt_2dt_1\right] \quad (4)$$

Since the above success modes are mutually exclusive, this yields:

$$R^{(3)}(t) \quad = \quad R_{\text{I}}(t) + R_{\text{II}}(t) + R_{\text{III}}(t) + R_{\text{IV}}(t)$$

Equation (2), (3), (4) and (5) are difficult to compute. The numerical integration technique using Simpson rule (Pan and Kolarik 1986) is proposed to solve the complicated computation involved in the above three equations.

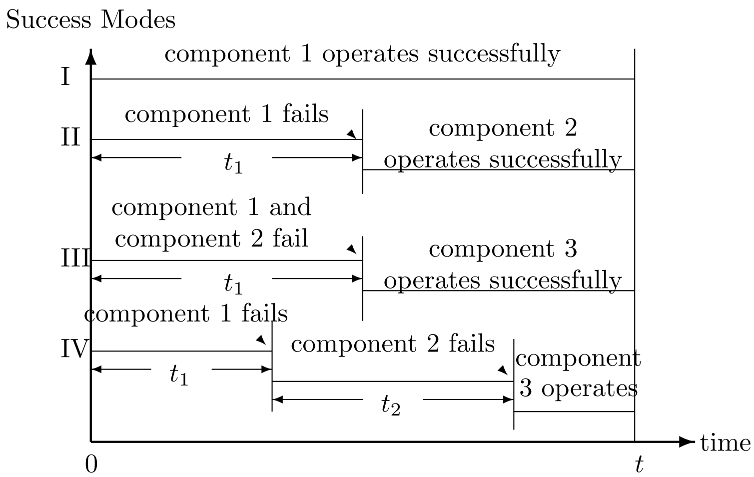

Figure 2: Success Modes of a three-component system

2.3 The multi-stage production system of m imperfect switching subsystems connected in series

Many modern equipments/systems are designed in a series configuration (as illustrated in Figure 3) with spares available when necessary. Assume the reliability function of each subsystem is $R_i(t), i = 1, \cdots, m$. Then the series system reliability for m subsystems connected in series can be written as $R(t) = \prod_{i=1}^{m} R_i(t)$

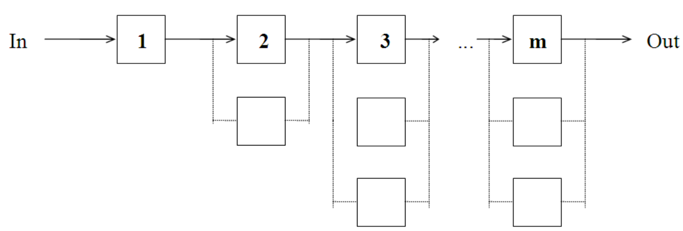

Figure 3: Series configuration of a m-stage production system

3 NUMERICAL EXAMPLE - A MANUFACTURING APPLICATION

Suppose one wants to predict the system reliability of a piece of automatic manufacturing equipment with four different machining tools (I, J, K, and L) and spares subject to Weibull failure (illustrated in Figure 4). The machining condition, speed, feed, and Weibull parameters of the tools as well as their associated costs based on reported tool life data (Pan and Kolarik 1986; Ramalingam et al. 1978) are summarized in Table 2. Given the information for one manufacturing batch, where the symbols P_1, P_2 and P_3 indicate different types of items need to be produced. The total machining time needed for each tool is given by I: 90+72+60=222 minutes, J: 22.5+57.5+100=180 minutes, K: 180+70+60=310 minutes, and L: 95+55=150 minutes.In this example, we also incorporate the total costs of spares required for each stage into the program and enumerate them following the associated system reliability so that the management can decide upon the desired system reliability at a given cost level. For example, using our developed computer algorithm, one can obtain predicted reliability of this alternative is 90.09%, where a total cost of $126.75 might be optimal.

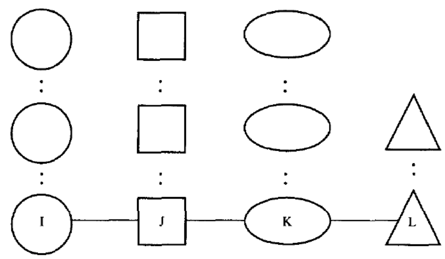

Figure 4: A multi-stage production system with four different machining tools and spares

Table 2: Tool information for interrupted machining

Tool type	speed	feed	α	β	cost($)
I	1.66	0.335	0.02400	0.50	8.85
J	1.17	0.125	0.00025	1.50	7.65
K	1.66	0.850	0.00075	1.10	9.95
L	1.66	0.265	0.00600	0.85	12.85

4 CONCLUSION

This paper has described a procedure to analyze the reliability of an imperfect multi-stage production system subject to Weibull failures. Such a technique is necessary to overcome the mathematical difficulties which will arise when failure distribution other than exponential is utilized. The accuracy and validity of the computer algorithm have been verified. Finally, a numerical example for manufacturing application has been given to illustrate the use of this recursive computer algorithm. With the advent of flexible manufacturing systems and other extremely high-cost machining systems, the trade-off between the cost of spares and system reliability predictions will be helpful to design and enhance the performance of future production systems.

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