Assessment of a control limit maintenance decision rule for a multi deteriorating mode system

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Abstract

This paper deals with a gradually deteriorating system which undergoes a change of its environment conditions. This change impacts the deterioration of the system and leads to an acceleration of its deterioration. The aim of this paper is to assess a control limit maintenance decision rule for such a system. The maintenance policy is then evaluated through its average long run cost rate which is computed, using semi regenerative properties, for different values of parameters.

1 Introduction

In this paper, a condition-based maintenance policy for a multi deteriorating mode system is evaluated using semi regenerative properties. The considered system gradually deteriorates and undergoes a change of its deterioration speed rate at an unknown time. This change can be due to environmental conditions, in which the system is evolving. The time of change of deterioration mode is a random variable with a known probability density function (pdf). To model the deterioration a gamma process is used as its deterioration is assumed to be strictly increasing (J.M. van Noortwijk 2009). The proposed condition-based maintenance policy is a control limit strategy in the framework of failure limit policy (H.Wang 2002). The aim of this paper is to assess the maintenance policy and to minimize its average long run cost rate. The system is described in section 2. The maintenance decision rules are exposed in section 3. An analytical evaluation of the maintenance policy is proposed in section 4 and section 5 deals with results from numerical experiments which illustrate the assessment.

2 System description

The considered system is an observable system which satisfies assumptions presented bellow.

2.1 General assumptions on the system:

- At time t, the deterioration level of the system can be summarized by a random variable Y_t. When no maintenance action has taken place, the process $(Y_t)_{t\geq 0}$ is an increasing stochastic process, with initial state $Y_0 = 0$. It means that the deterioration of the system is strictly increasing and that at time $t = 0$, the system is new.
- The system is said to be "failed" if its deterioration level reaches a predetermined threshold L, i.e. if $Y_t \geq L$. This threshold L is chosen in respect with the intrinsic properties of the system and it can be seen as a safety level which has not to be exceeded. When the deterioration level exceeds the safety level, the system continues to deteriorate until the failure is detected and the system is repaired.
- To get information on the deterioration level of the system and to be able to detect failures, the system is inspected periodically. The time between two inspections is called the inter-inspection time and is denoted by Δt.

2.2 Specific assumptions on the system:

- The mean deterioration of the system increases after an unknown time represented by the random variable T_0 with a known pdf denoted by f_{T_0}. At a realization of T_0 it is considered that there is a change of deterioration mode. Before the change of mode, the system is deteriorating according to a nominal mode M_1 and after the change of mode, the system is deteriorating according to an accelerated mode M_2.

- In the case of a single deteriorating mode system, future deterioration is independent of the current level of deterioration and depends only on the period over which the system is allowed to deteriorate because of the independence and stationarity of increments.
- It is assumed that the deterioration process in mode M_i, denoted by $(Y_t^i)_{t\geq 0}$, $(i = 1, 2)$, is a gamma process as it satisfies the properties of the deterioration of the considered system. For all $0 \leq s < t$, increments of $(Y_t^i)_{t\geq 0}$ between s and t, $Z_{t-s}^i = Y_t^i - Y_s^i$, follow a gamma probability density function with shape parameter $\alpha_i.(t-s) \in \mathbb{R}^{+*}$ and scale parameter $\beta_i \in \mathbb{R}^{+*}$, expressed as follow:

$$g_{\alpha_i.(t-s),\beta_i}(z) = \frac{1}{\Gamma(\alpha_i.(t-s))} \cdot \frac{z^{\alpha_i.(t-s)-1} e^{-\frac{z}{\beta_i}}}{\beta_i^{\alpha_i.(t-s)}} \mathbb{I}_{\{z\geq 0\}}. \tag{1}$$

- Parameters of the nominal deterioration mode M_1 are (α_1, β_1) and parameters of the accelerated deterioration mode M_2 are (α_2, β_2). Moreover, $\alpha_2\beta_2 > \alpha_1\beta_1$ where $\alpha_i\beta_i$, $(i = 1, 2)$ is the mean deterioration per time unit.
- If a change of deterioration mode of the system has occurred, it is assumed that it is perfectly detected through inspections.

3 Maintenance policy: Decision rules

The maintenance policy is driven by the knowledge of the system state after inspections. As it is specified in section 2, all information which can be collected through inspections are: the real deterioration level $y \in \mathbb{R}^+$, the mode of deterioration of the system $i = 1, 2$ and the number of inspections already performed $j \in \mathbb{N}$ since the system is new. Hence, the system state is summarized by the triplet (i, j, y).
In order to avoid failures a preventive maintenance action is performed. At each inspection time, a decision has to be made, whether a maintenance action should be taken, or whether the system should be left as it is until the next inspection. The two possible maintenance actions that can be performed at each inspection time are preventive or corrective replacements. Each maintenance operation is assumed to be perfect (the system is "as good as new" after maintenance) and instantaneous. Let M be the preventive threshold, whatever deterioration mode. The possible decisions which can arise at each inspection time t_j are defined as follow:

- If $Y_{t_j} < M$ then the system is left as it is until the next inspection.
- If $M \leq Y_{t_j} < L$ the system is too badly deteriorated so it is preventively replaced.
- If $Y_{t_j} \geq L$ the system is considered as failed and it is correctively replaced.

4 Evaluation of the maintenance policy

To evaluate the maintenance policy proposed in section 3, the average long run cost rate $\mathbb{E}[C_\infty]$ is computed. Let us denote by C_c (respectively C_p) the cost of a corrective (respectively a preventive) maintenance operation and the cost incurred by any inspection is C_{insp}. In the period of unavailability of the system denoted $d_u(t)$ an additional cost per unit of time C_u is incurred.
The global maintenance cost at a time t is given by the following equation:

$$C(t) = C_{insp} \cdot N_{insp}(t) + C_p \cdot N_p(t) + C_c \cdot N_c(t) + C_u \cdot d_u(t), \tag{2}$$

where $N_{insp}(t)$ is the number of inspections before t, $N_p(t)$ (respectively $N_c(t)$) is the number of preventive (respectively corrective) actions before t. As the criterion of evaluation is the average long run cost rate $\mathbb{E}[C_\infty]$, using equation (2), it can be expressed as follow:

$$\mathbb{E}[C_\infty] = \lim_{t\to\infty} \frac{\mathbb{E}[C(t)]}{t}. \tag{3}$$

After each inspection the evolution of the system state (i, j, y) depends only on the system state at that time. The information on the change of deterioration mode (i and j) is important, because if the change hasn't occur yet, i.e. $i = 1$, $\mathbb{P}[t_0 \leq t_{j+1}|t_0 > t_j] \geq \mathbb{P}[t_0 \leq t_j|t_0 > t_{j-1}]$ where t_0 is a realization of the random variable T_0. The process $(i, j, Y_t)_{t\geq 0}$ is then a semi-regenerative process with semi-regeneration times t_j (inspection times). The propriety of the semi-regeneration process allows us to write:

$$\mathbb{E}[C_\infty] = \lim_{t\to\infty} \frac{\mathbb{E}[C(t)]}{t} = \frac{\mathbb{E}_\pi[C(\Delta t)]}{\Delta t}, \tag{4}$$

where π is the stationary measure representing the pdf of the maintained system states and $\mathbb{E}_\pi[C(\Delta t)]$, the average value of $C(\Delta t)$ under the stationary measure π defined by the following equation:

$$\mathbb{E}_\pi[C(\Delta t)] = C_{insp} \cdot \mathbb{E}_\pi[N_{insp}(\Delta t)] + C_p \cdot \mathbb{E}_\pi[N_p(\Delta t)] + C_c \cdot \mathbb{E}_\pi[N_c(\Delta t)] + C_u \cdot \mathbb{E}_\pi[d_u(\Delta t)]. \tag{5}$$

4.1 Stationary measure π

To be able to compute the cost in the semi-regenerative frame, a stationary measure π is needed. This stationary measure depends on the deterioration mode M_i, $(i = 1, 2)$, the number of inspections j performed since the last maintenance action and the deterioration level y observed after the j^{th} inspection. Considering those information, it is possible to give a general expression of $\pi(i, j, dy)$. In extrapolating the general expression in a single mode deteriorating system given by (A.Grall, L.Dieulle, C.Berenguer, and M.Roussignol 2002), the general form of the stationary measure can be expressed as follow:

$$\pi(i, j, dy) = a\delta_0(dy)\mathbb{I}_{\{i=1,j=0\}} + (1-a)\Big(b_1(j, y)dy\mathbb{I}_{\{i=1,j>0\}} + b_2(j, y)dy\mathbb{I}_{\{i=2,j>0\}}\Big) \tag{6}$$

with $0 < a < 1$ and b_1 and b_2 two pdf on $[0, M)$. π is solution of the following invariance equation:

$$\pi(i, j, dy) = \sum_{l=1}^{2} \sum_{k=0}^{+\infty} \int_0^M \mathcal{T}(i, j, dy|l, k, x)\pi(l, k, dx), \tag{7}$$

where $\mathcal{T}(i, j, dy|l, k, x)$ is the transition rate from state (l, k, x) to state (i, j, y). Considering stationarity and independence of increments of gamma process, the system state at the j^{th} inspection, (i, j, y) depends only on the system state at the $(j-1)^{th}$ inspection $(l, j-1, x)$. It is then possible to find the following expression of b_1 and b_2 using equations (6) and (7):

$$b_1(j, y)\mathbb{I}_{\{i=1,j>0\}} = \frac{a}{1-a}\overline{F}_{T_0,0}(\Delta t)g_{\alpha_1\Delta t,\beta_1}(y)\mathbb{I}_{\{j=1\}} + \overline{F}_{T_0,j-1}(j\Delta t)\int_0^y b_1(j-1, x)g_{\alpha_1\Delta t,\beta_1}(y-x)dx\mathbb{I}_{\{j>1\}}, \tag{8}$$

$$\begin{aligned} b_2(j, y)\mathbb{I}_{\{i=2,j>0\}} &= \frac{a}{1-a}\int_0^{\Delta t} f_{T_0,0}(t_0)\Big(g_{\alpha_1 t_0,\beta_1} * g_{\alpha_2.(\Delta t - t_0),\beta_2}\Big)(y)dt_0\mathbb{I}_{\{j=1\}} \\ &+\Bigg(\int_0^y b_1(j-1, x)\int_{(j-1)\Delta t}^{j\Delta t} f_{T_0,j-1}(t_0)\Big(g_{\alpha_1.(t_0-(j-1)\Delta t),\beta_1} * g_{\alpha_2.(j\Delta t - t_0),\beta_2}\Big)(y-x)dt_0dx \\ &\qquad + \int_0^y b_2(j-1, x)g_{\alpha_2\Delta t,\beta_2}(y-x)dx\Bigg)\mathbb{I}_{\{j>1\}}, \end{aligned} \tag{9}$$

where $f_{T_0,j}$ is the pdf of the time change of mode given j, $F_{T_0,j}$ the cumulative density function of $f_{T_0,j}$, $g_{\alpha_i t,\beta_i}$ a gamma pdf with parameters $(\alpha_i t, \beta_i), (i = 1, 2)$ and $*$ the convolution operator.

4.2 Numbers of repairs, inspections and period of unavailability

In a semi-regenerative frame, $\mathbb{E}_\pi[N_{inps}(\Delta t)] = 1$. Other quantities such as $\mathbb{E}_\pi[N_p(\Delta t)]$, $\mathbb{E}_\pi[N_c(\Delta t)]$ and $\mathbb{E}_\pi[d_u(\Delta t)]$ need to be computed. The stationary measure $\pi(.)$ is expressed as follow:

$$\pi(.) = \sum_{i=1}^{2} \sum_{j=0}^{+\infty} \pi(i, j, .), \tag{10}$$

where $\pi(.)$ represents the pdf of the maintained system states. The pdf of the deterioration of the system is expressed as follow:

$$f_{\Delta t}(x, y) = \underbrace{g_{\alpha_1\Delta t,\beta_1}(y-x)\mathbb{I}_{\{x_{t_0}>y\}}}_{mode\ 1} + \underbrace{g_{1,2,\Delta t,t_0}(x, y)\mathbb{I}_{\{x<x_{t_0}<y\}}}_{change\ of\ mode} + \underbrace{g_{\alpha_2\Delta t,\beta_2}(y-x)\mathbb{I}_{\{x_{t_0}<x\}}}_{mode\ 2}, \tag{11}$$

where x_{t_0} represents the deterioration level of the system at the time of change of mode t_0 and $g_{1,2,\Delta t,t_0}$ a convolution of two gamma pdf ($g_{\alpha_1\Delta t,\beta_1}$ and $g_{\alpha_2\Delta t,\beta_2}$). $f_{\Delta t}$ represents the density of deterioration levels during Δt given the initial deterioration level x.

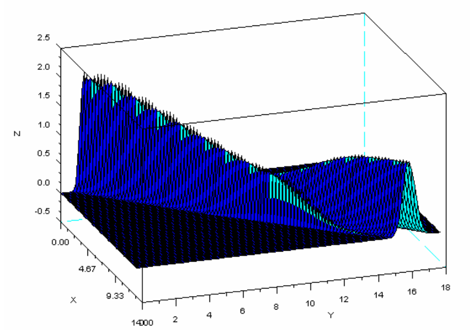

Figure 1

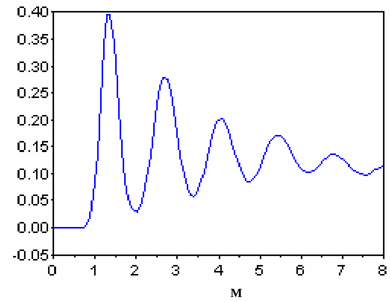

Figure 2

In a semi-regenerative frame, $\mathbb{E}_\pi[N_p(\Delta t)]$ represents the probability that the deterioration level of the system is in between M and L between two inspections, hence:

$$\mathbb{E}_\pi[N_p(\Delta t)] = \mathbb{P}_\pi(M \leq Y_{t,t\in[0;\Delta t]} < L) = \int_0^M \left(\int_M^L f_{\Delta t}(x,y)dy \right) \pi(dx). \quad (12)$$

In the same way, the number of corrective repairs and the period of unavailability are given by the following equations:

$$\mathbb{E}_\pi[N_c(\Delta t)] = \mathbb{P}_\pi(Y_{t,t\in[0;\Delta t]} \geq L) = \int_0^M \left(1 - \int_0^L f_{\Delta t}(x,y)dy \right) \pi(dx), \quad (13)$$

$$\mathbb{E}_\pi[d_u(\Delta t)] = \int_0^M \left(\int_0^{\Delta t} \left(1 - \int_0^L f_s(x,y)dy \right) ds \right) \pi(dx). \quad (14)$$

5 Numerical implementation

The numerical implementation is done on a real case of study. The application is based on data from (C.Meier-Hirmer, F.Sourget, and M.Roussignol 2005) which model the deterioration of a French High Speed Train line connecting Paris and Lyon. The parameters of deterioration used for the numerical implementation are $(\alpha_1; 1/\beta_1) = (0.288; 31.76)$, $(\alpha_2; 1/\beta_2) = (0.521; 24.53)$ and $L = 13$. The maintenance costs are $C_{insp} = 25$, $C_p = 5000$ and $C_c = C_u = 580000$. Results obtained are given in figures 1 and 2. Figure 1 represents the pdf of the deterioration of the system on $\Delta t = 150$ and figure 2 represents the stationary law for $\Delta t = 150$ and $M = 8$. To simplify the computation of the cost, the period of unavailability has been computed considering a single deteriorating mode (with M_2). In this case, for $\Delta t = 150$ the cost is minimized for $M = 8,5$ and it is equal to $8,175$. Numerical problems did not allow us to find an optimal couple $(\Delta t, M)$ yet.

6 Conclusion

The aim of this paper is to assess the condition-based maintenance policy for a system which gradually deteriorates according to two modes of deterioration. For such a study, the stationary law and the probability density function modeling the system deterioration between two inspections are developed. Those analytical results have been applied on a real case of study and the preventive threshold which minimizes the average long run cost rate of the maintenance policy for a given inter-inspection time has been found. Some numerical problems to compute the cost did not allow us to optimize the maintenance policy yet.

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