The Reparability Function: Further Steps toward the Science of Reliability

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Abstract

This research has multiple goals. As first I calculate the ability of working (= *effectiveness*) of systems and their resistance to repairs (= *reparability*) that are systemic features often treated in qualitative terms. I suggest the use of the Boltzmann-like entropy to derive the *reparability function* that relates the above mentioned properties exhibited in parallel by a system during the whole lifespan. This paper pursues a second scope. The theory of reliability has not yet progressed as a science since most of the researches are grounded upon pure statistical models. Inductive-experimental approaches are more popular than deductive-rational approaches. Instead the present research derives the behavior of a system on the basis of 'physical' modeling and proves to be in line with the style typical of mature disciplines.

1 Introduction

Frequently writers describe verbally what the effectiveness of a system is, the reparability and so forth. Theorists refer the results of a repair to two levels of quality named 'good-as-new' and 'bad-as-old' [1] and these concepts are associated to 'high' and 'low' values of probability of the functioning state respectively but the boundaries for such probability ranges did not find universal consensus.

Efforts to define the general laws through 'physical' models may be found since the seminal works in the reliability field [2], [3] but the progression turns out to be slow. Most studies in the field develop pure statistical models. I attempted to append my contribution to this line of research and introduced the *Boltzmann-like* or *stochastic entropy* $H(P_i)$ to quantify some systemic features [4]. I also used this entropy to derive the probability of good-functioning during the life-maturity and aging using appropriate 'physical' structures [5]. The present paper formalizes the abilities to work and to be repaired using the entropy function, and proposes an accurate analysis of the 'good-as-new' and 'bad-as-old' states. In this way the introduction of inferential reasoning attempts to mature the reliability theory as a logical-deductive discipline.

2 The Reparability Function

Notation

$S(A_f, A_r)$ is a stochastic system.

A_f is the functioning or steady state during which the system runs.

A_r is the recovery state during which the system is repaired, or renewed etc.

P_i is the probability of the generic state i.

$H_i = f(P_i) = k_1 ln(P_i) + k_2$ is the Boltzman-like or stochastic entropy of the state A_i.

$H_f = f(P_f)$ is the entropy of A_f named reliability entropy.

$H_r = f(P_r)$ is the entropy of A_r named recovery entropy.

I introduce the reliability entropy and the recovery entropy in order to investigate the behavior of repaired systems

$$\begin{aligned} H_f &= a_1 \ln(P_f) + a_2 \\ H_r &= b_1 \ln(P_r) + b_2 \end{aligned} \tag{1}$$

Where a_1, a_2, b_1, b_2 are constant typical of S. Because A_f and A_r are mutually exclusive, we have the ensuing constraint

$$P_f + P_r = 1 \tag{2}$$

For the sake of simplicity I assume $a_1 = b_1 = 1$ and $a_2 = b_2 = 0$. From (1) and (2) I obtain the reliability entropy in function of the recovery entropy and I call this result as the *reparability function*

$$\widetilde{H}_f = H_f(H_r) = \ln(1 - e^{H_r}) \tag{3}$$

This function (Figure 1) has the horizontal asymptote $H_f = 0$ and the vertical asymptote $H_r = 0$. The elasticity $\varepsilon(f)$ provides the accurate description of the function $f(x)$ uninfluenced by the scales of the axes. The elasticity function of (3) is

$$\varepsilon(\widetilde{H}_f) = -\frac{H_r e^{H_r}}{(1 - e^{H_r})\ln(1 - e^{H_r})} \tag{4}$$

Elasticity increases linearly for low values of H_r and increases very fast for H_r close to zero (Figure 2). The elasticity function inverts trend approximately within the ordinate range $(-1, -0.5)$, this range corresponds to the rounded values $(-0.69, -0.17)$ in the H_r axis. In conclusion one can define the extremes of two precise zones: $\alpha = (-\infty, -0.69), \beta = (-0.17, 0.00)$ which are separated by the intermediate range $\gamma = (-0.69, -0.17)$. Function (3) decreases linearly in α and decreases rapidly in β.

If a_1, a_2, b_1, b_2 are any, I obtain the general version of the reparability function from (1) and (2)

$$H_f = H_f(H_r) = a_1 \ln(1 - e^{\frac{H_r - b_2}{b_1}}) + a_2 \tag{5}$$

The constant a_2 and b_2 determine the position of function (5) in the Cartesian space, while a_1 and b_1, determine the scales of the axes (Figure 2). Curve (5) has the same structure as (3) since elasticity (4) describes the reparability function independently from the factors of the scales and the shift of the curve. I find the asymptotes $H_f = a_2$ and $H_r = b_2$ from the limits

$$\begin{aligned} \lim_{H_r \to b_2} H_f &= -\infty \\ \lim_{H_r \to -\infty} H_f &= a_2 \end{aligned} \tag{6}$$

The typical ranges for α and β are redefined using (5)

$$\alpha = \Big(-\infty, a_1 \ln(1 - e^{\frac{-0.69 - b_2}{b_1}}) + a_2\Big)\ ;\ \beta = \Big(a_1 \ln(1 - e^{\frac{-0.17 - b_2}{b_1}}) + a_2, b_2\Big) \tag{7}$$

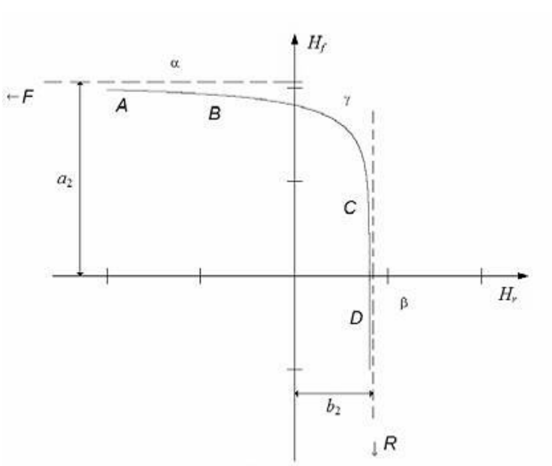

Figure 1:

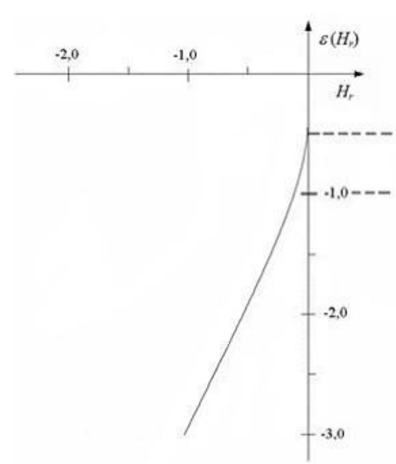

Figure 2:

3 Physical Interpretation

This section comments on the physical meanings of the foregoing results and on the advantages of the present formalization.

3.1 Remark — *Physical model*

Assumption (2) means that the states A_f and A_r are mutually exclusive

$$S = S(A_f, A_r) = S(A_f \text{ OR } A_r) \tag{8}$$

In principle a dynamic system has also the idle state A_k during which the system is good and idles, and the *dereliction state* A_w during which the system is broken and is not manipulated. Totally there are four mutually exclusive states: A_f, A_r, A_k, and A_w. The states A_k and A_w are static and do not contribute to the system evolution, A_k and A_w are null for the present inquiry which focuses on reversible/ irreversible phenomena and we obtain the model (8).

Machines stop when they are repaired; A_f and A_r do not overlap and physical model (8) appears self-evident for artificial devices. By contrast living systems do not stop to live when they are cured and assumption (8) is more delicate in the biological domain. The state A_r may be classified in the following way:

1) *Chronic* or *long-term diseases* are to be associated to the progressive decay of systems already considered in the present theory [5].
2) The state A_f shows reduced performances during *medium-term diseases.* Sometimes even important metabolic functions retard or even break off during a therapy, for example the growth of a child can stop due to a heavy disease [6]. Thus the state A_f may be considered as negligible and eqn (8) is reasonable in the present study.
3) During a *short-term disease* or a *feeble disease*, the superposition of the states appears small in amount respect to the whole lifespan [7]. The effects of A_r are not significant and A_r is approximately null. Eqn (8) is acceptable also in the present event.

This analysis yields that physical model (8) is generally true in the world.

One may object that the stochastic system is not new in literature.

The novelty of the present work lies in the method to calculate the stochastic system which uses the entropic function.

3.2 Remark – *The physical significance of H_f and H_r*

In fact the concept of reversibility/ irreversibility of a state spells the physical ability of S in that state, notably when A_f is rather irreversible, it means that S works steadily, conversely when A_f is reversible, the functioning state is unstable and S is unreliable in the world [4], [5]. Symmetric reasoning yields that the recovery entropy gives the ability/inability of S to be restored to operating condition, in particular when A_r is rather irreversible, it means that S is hard to repair. Reversible A_f denotes an easily reparable system. One reasonably conclude that H_f and H_r calculate the *effectiveness* and the *reparability* of systems respectively.

3.3 Remark – *The reparability function*

As a consequence eqn (5) relates the *effectiveness* and the *reparability*, and proves how a steady system is easy to repair and a bad functioning system is hard to repair. Universal experience shows how this theoretical result is true for devices and machines, and is also valid for living systems which are easily healed when they live in good health and are cured with difficulties when individuals live in decrepit conditions.

The reparability function provides a novel viewpoint on the reliability phenomenology because $H_f(H_r)$ establishes a time-independent rule. The physical features of S are not studied respect to the time as the bathtub curve does, instead the *effectiveness* and the *reparability* depend each other and (5) does not rise the criticism of the hazard function that mismatches with some experimental temporal trends [8].

Extremes F and R denote systems perfectly functioning and systems absolutely disrupted. These extremes constitute two unachievable cases in the present framework, thus they are ideal cases, similar to the cases studied in mature disciplines such as mechanics, hydraulics and electronics. For example the *movement without attrition* is an ideal case in mechanics. This movement is the world nonetheless is a fundamental reference in the mechanics calculations as an experiment may get more or less close to this ideal case. Symmetrically a physical system can come more or less near F and R. One concludes that the zones α and β adjacent to F and R respectively correspond to the physical states termed with 'good-as-new' and 'bad-as-old'.

In current literature these states are generically assigned with the probability of good functioning $P \sim 1$ and $P \sim 0$ respectively, instead the present theory relates the following properties for 'good-as-new' and 'bad-as-old'. Suppose $\Delta H_r = const$ is a repair/maintenance of S:

i) If S is placed in β the intervention ΔH_r causes the finite increase of performances $\Delta H_f = M$.

ii) If S lies in α, the intervention ΔH_r causes the finite increase of performances $\Delta H_f = m$.

Where

$$m \ll M \tag{9}$$

This result may be illustrated in practical terms. Let the intervention $\Delta H_{rx} = const = k$ substitutes the component x; if the renewal occurs in β, the new component takes the place of a wear-out piece and the improvement M is noteworthy. If the renewal occurs in α, x takes the place of a nearly new piece and the benefit m is negligible.

The finite progress m consists with the definition of F which is an ideal case and cannot be reached through a finite number of steps. Concluding, the states α and β have lower and upper bounds as well and have precise physical properties.

4 Conclusions

The present inquiry adopts the stochastic entropy to quantify two systemic characters: the *effectiveness* H_f and the *reparability* H_r. The reparability function $H_f = f(H_r)$ have been deduced and in particular we have discussed the ideal cases F and R and the states usually marked with the terms 'good-as-new' and 'bad-as-old' that have precise boundaries and physical properties.This is the first contribution of the present work.

Results from (4) to (7) match with current literature but they are not trivial because the present study derives those results from (1) and (8), and follows a deductive pathway. That is to say the present study puts in practice the inferential methodology. In this way I attempt to provide a further contribution to the migration of the reliability theory toward a mature discipline. In fact the followers of a discipline argue from a given model or from known axioms to make statements about samples drawn from a known population. Deductive methods should be introduced into the reliability theory to transform this theory into a thorough discipline and to complete the inductive methods developed so far.

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