Stochastic modelling of cylinder liners wear in a marine diesel engine

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Abstract

Ship Diesel engines are requested to have high reliability and availability levels. A major factor in determining failure of heavy-duty Diesel engines is the ring/liner wear. In high power Diesel marine propulsion engines maximum wear usually occurs in the top region of the cylinder liner, which is subject to high thermomechanical and tribological stresses that produce relevant early local damages.

This paper presents a stochastic model of cumulative damage that can be used to model the wear process of the cylinder liners of marine naval Diesel engines. The model could be used to perform condition based reliability estimation and to plan condition based maintenance activities.

1 Introduction

We present and discuss a stochastic model describing the wear process of cylinder liners in a marine diesel engine. Our interest is motivated by actual data, provided by a leading Italian ship company, operating freight and passengers ships, both in Europe and overseas.

The interest in the liner wear arises since it is one of the major factor in determining failure of heavy-duty diesel engines, which are used in ships and are requested to have high reliability and availability levels. In high power diesel marine propulsion engines maximum wear usually occurs in the top region of the cylinder liner, which is subject to high thermomechanical and tribological stresses that produce relevant early local damages.

According to many studies on diesel engine wear, it seems that the major wear in this region is caused by the high quantity of abrasive particles on the piston surface, occurring by the combustion of heavy fuels and oil degradation (soot). The soot abrasive wear mechanism acts when the lubricant film thickness is less than the soot particle size, so that the soot is involved in a three-body abrasive action with the liner metal surfaces on one side and the piston surface on the other. The wear of the liner occurs because the soot particles are harder than the corresponding engine parts.

In addition to abrasive wear, a corrosive wear has also been observed, and it is due to sulphuric acid, nitrous/nitric acids and water. Physical considerations lead to identify a unique place in the liner, called Top Dead Center, in which almost all failures occur once wear exceeds a specified threshold. Therefore, measurements are performed using a micrometer near the Top Dead Center. The paper presents a stochastic model about the time evolution of the wear process of the cylinder liners of marine naval diesel engines, as measured by the thickness of the liners. The model is based on a stochastic differential equation and it can be used to perform condition based reliability estimation and to plan condition based maintenance activities. As soon the predicted probability of a given threshold wear exceeds a given level at some future time t, then the ship is to be stopped and inspected to check the actual wear. The relevant economic aspects are about the costs of stopping ships for inspection and change of the huge liners (approximately 10 meters high) and the warranty clauses which make the ship owner responsible when failures occur when the wear exceeds a threshold, set at 4 millimeters. In the paper, we concentrate on the mathematical aspects of the model, and on the motivations leading to the model and its critical discussion.

2 Assumptions

Empirical studies of the wear and the physical properties of the liner lead previous studies, e.g. (Bocchetti, Giorgio, Guida, and Pulcini 2006), to observe that wear increment decreases as a function of wear. Furthermore, background activity of *tiny* particles leads to almost negligible wear increments, whereas *large* particles are responsible for the most relevant wear increments. Instead of considering the wear $W(t)$, we prefer to model the thickness $T(t) = 100mm - W(t)$ of the liner's wall.

From a mathematical viewpoint, we consider a stochastic differential equation (SDE) $dT(t) = f(T(t))$ to relate the evolution of the thickness $T(t)$ over time and its influence on the thickness decrement $dT(t)$. Furthermore, thickness reduction, caused by a sequence of small normally distributed effects, due, e.g., to the *tiny* particles or corrosion, makes reasonable the choice of a Brownian motion with drift, where the latter is the mean value of the process. Finally, we consider thickness increment jumps due, possibly, to *large* particles, i.e. the soot, and we model them using a jump process in the SDE.

3 Stochastic model

Based on the assumptions of Section 2, we suppose that the evolution of the thickness process $T(t)$ is

$$dT(t) = -T(t^-) \left\{ \mu dt + \sigma dB(t) + dJ(t) \right\}, \tag{1}$$

where μ and σ are constants, B is a standard Brownian motion and J is a process independent of B with piecewise constant sample paths. The symbol $dJ(t)$ in (1) stands for the jump in J at time t. In particular, J is given by

$$J(t) = \sum_{j=1}^{N(t)} Y_j, \tag{2}$$

where $Y_1, Y_2, \ldots$ are i.i.d. random variables and $N(t)$ is a homogeneous Poisson process (HPP) with rate $\lambda > 0$ that counts the number of collisions of *large* particles up to time t; we use $N(s,t)$ for the number of collisions in $(s,t]$. Let $\{\tau_j\}_{j\in\mathbb{N}}$ be the sequence of random arrival times of $N(t)$, that is, the (unobserved) collision times. From (1) and (2), it follows that the relative jump size is

$$Y_j = \frac{T(\tau_j^-) - T(\tau_j)}{T(\tau_j^-)},$$

at each τ_j, for every j. With this assumption, we restrict Y_j to be strictly positive random variables and hence we ensure T can never become negative.

We consider a lognormal distribution for the relative jumps, i.e. $(Y_j + 1) \sim \mathcal{LN}(a, b^2)$, for every $j \geq 1$. It is worth mentioning that $Y_j + 1 > 1$ and that $\log(Y_j + 1) > 0$; nonetheless, we consider the lognormal distribution since, as in many other works, the parameters a and b^2 are chosen such that the probability of negative values of $\log(Y_j + 1)$ will be negligible under its Gaussian distribution. We set t_0 as the starting time.

The proposed model has many positive aspects; first of all, there exists a closed-form solution of the SDE and it is possible to provide an explicit expression for the conditional likelihood, upon the number of collisions in each interval between inspection times. The latter aspect will be very useful in Bayesian inference since the MCMC (Markov chain Monte Carlo) algorithm will rely on the conditional likelihood.

The solution of the SDE (1), see (Runggaldier 2003), is given by

$$T(t) = T(0) \exp\left\{ -(\mu + \sigma^2/2)t - \sigma B(t) \right\} \prod_{j=1}^{N(t)} (1 - Y_j) \tag{3}$$

Conditional upon $N(t) = n$ and taking $t_0 = 0$, (3) becomes

$$\begin{aligned} T(t) &\sim T(0) \cdot \mathcal{LN}\left(-(\mu + \sigma^2)t, \sigma^2 t\right) \cdot \mathcal{LN}\left(an, b^2 n\right) \\ &\sim \mathcal{LN}\left(\log T(0) - (\mu + \sigma^2/2)t + an, \sigma^2 t + b^2 n\right) \end{aligned} \tag{4}$$

Starting at s, with $T(s) > 0$, and conditional on $N(s,t) = n$, it follows that

$$T(t) \sim \mathcal{LN}\left(\log T(s) - (\mu + \sigma^2/2)(t-s) + an, \sigma^2(t-s) + b^2 n\right. \tag{5}$$

Since the increments are independent ((Øksendal 1998)), it follows that
$f(T(t_n) = w_n, \ldots, T(t_1) = w_1) =$

$$\begin{aligned}
&= f(T(t_n) - T(t_{n-1}) = w_n - w_{n-1}, \ldots, T(t_1) - T(t_0) = w_1 - w_0) \\
&= \prod_{i=1}^{n} f(T(t_i) - T(t_{i-1}) = w_i - w_{i-1}) \\
&= \sum_{n_1,\ldots,n_n} \prod_{i=1}^{n} f(T(t_i) - T(t_{i-1}) = w_i - w_{i-1} | N(t_{i-1}, t_i) = n_i) f(N(t_{i-1}, t_i) = n_i) \qquad (6)
\end{aligned}$$

Inferences based on the likelihood (6) are too difficult to deal with, since they need specification of all possible $f(N(t_{i-1}, t_i) = n_i$ and summation over all of them.

The current research is focused on the partial likelihood

$$\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2(t_i - t_{i-1}) + b^2 n_i} w_i} \exp\{-\frac{[\log w_i - \log w_{i-1} + (\mu + \sigma^2/2)(t_i - t_{i-1}) - a n_i]^2}{2[\sigma^2(t_i - t_{i-1}) + b^2 n_i]}\}$$

based on the knowledge of $N(t_{i-1}, t_i) = n_i$, $i = 1, \ldots, n$. The Bayesian approach allows for specification of such quantities, by treating $N(t_{i-1}, t_i) = n_i$, $i = 1, \ldots, n$, as *parameters* and drawing from their (conditional) posterior distribution in the MCMC sampling.

Discussion

We have proposed a stochastic model which describes the evolution of the thickness of the walls in cylinder liners. We are currently implementing Bayesian models which are treating the number of jumps in each interval as latent variables which can be treated as parameters in the Markov chain Monte Carlo simulation. The Bayesian models differ for the way the parameters are treated: equal, distinct or exchangeable for all engines.

References

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