Reliability Model for Hierarchical Systems*

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Abstract

Most of real technical systems and biological objects with sufficiently high organization are complex hierarchical partially controllable systems. In the paper for modeling and analysis of reliability of such systems the methods of decomposable semi-regenerative processes is used. Some simple example illustrate our approach.

1 Introduction and Motivation

In terms of reliability, most of technical systems and biological objects with sufficiently high organization are complex hierarchical partially controllable systems. The failures in the systems of this type arise as a result of stress accumulation of the lowest (elementary) level, which pass several stages before the total failure. These faults lead to the decreasing efficiency of the system but do not lead to the total failure of the system. The system of control (SoC) fixes these fault stages of elements and gives the signal about the system "state of health" decreasing. According to these signals appropriate mechanisms of self-regulation are "switched on", and the system is self regenerated if the process disturbing is not too deep. In the last case some external action is needed. It is supposed that this action being applied in time and in needed quality and quantity turns the system after some time to the normal functioning state. In another case the delay with maintenance of the system leads to the system degradation and as a result it leads to the total failure of the system. For biological systems, for instance, the neuron system plays a role of controlling system, and it possesses high reliability. This means that biological objects can be treated as complex hierarchical controllable fault-tolerant reliability systems. For different technical systems there exist analogous high reliable systems of control. We focus on the study of the survival function, because it is the main characteristic for biological objects and complex technical systems.

In some previous papers we considered such type of models under Markov assumptions [1] - [2]. In this paper we propose a general mathematical model for the description and evaluation of the survival function of complex hierarchical systems with general distributions of units life times as well as the repair times of failed units, subsystems and the whole system.

There are several approaches to model the reliability of systems with general life- and repair times distributions. However, anyhow all of them are reduced to markovization of the process that describes the system behavior. One of them was proposed by Yu.K. Belyaev [3], and consists in construction of so-called linear-wise Markov processes. Another approach was developed in the works of N.P. Buslenko, I.N. Kovalenko and V.V. Kalashnikov [4] - [9], who proposed and elaborated the mathematical technique for the study of so-called piecewise linear aggregative systems. The further development of this theory was done in the works [10] - [15], where the notion of Decomposable Semi-Regenerative Processes (DSRP) was proposed and methods for its investigation were developed. In this paper these methods are applied to the investigation of the reliability of complex hierarchical systems.

2 General Model

Consider a complex hierarchical multi-component system which is controlled and managed by a high-reliable system of control (SoC), shown in fig 1. Assume that the system is constructed from blocks and branches of several levels. Each block and the following after branches and blocks forms a hierarchical subsystem of the same type as the main one. The blocks of the lowest level will be referred to as units and they are subjected to gradual failures of their own type. For the simplicity we limit ourselves to the binary systems only. We will denote by L the maximal level of units, and it is not necessary that any unit belongs to this level. Units of different levels are possible. The reliability of each unit is partially controllable.

In order to specify the description of its behavior we introduce a vector index $\mathbf{i} = (i_1, i_2, \ldots, i_{L_i})$ which determines each unit of the system as belonging to an appropriate chain of blocks at any level. Denote by $\mathcal{I}$ the set of these indices (and appropriate units). Then the state space of the system can be represented as $\mathcal{E} = \{\mathbf{j} = (j_{\mathbf{i}} : i \in \mathcal{I})\}$, where for any $\mathbf{i} \in \mathcal{I}$ the integer $j_{\mathbf{i}}$ represents the state of the $\mathbf{i}$-th unit in sense of its reliability. To specify the subsystem of k-th level we will refer to it as $\mathbf{i}^k = (i_1, \ldots, i_k)$ and the states of the appropriate subsystem will be denoted by ${\mathbf{j}_{\mathbf{i}}}^k$.

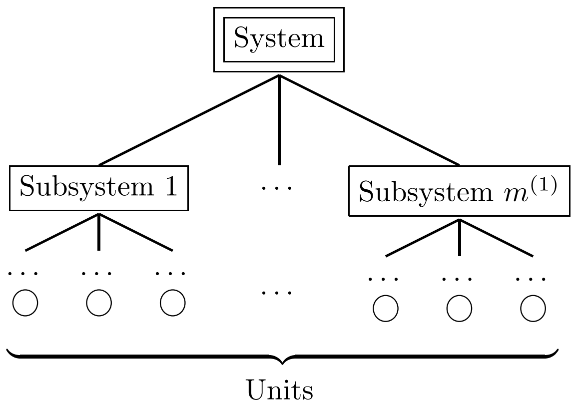

Figure 1: A complex multi-level hierarchical system.

It is supposed that the life times of units as well as the repair times of failed units, subsystems and the whole system are independent random variables (r.v.) that have general distributions. These r.v. may depend on the type of the unit. We denoted by $A_{\mathbf{i}}$ and $B_{\mathbf{i}_k}$, respectively, their appropriate cumulative distribution functions (c.d.f.) which are given by

$$A_{\mathbf{i}}(x) = \mathbf{P}\{A_{\mathbf{i}} \le x\}, \qquad B_{\mathbf{i}_k}(x) = \mathbf{P}\{B_{\mathbf{i}_k} \le x\}.$$

3 Investigation of the Reliability of a General Model

According to the assumptions above we model the reliability of such a system by multi-dimensional process $\mathbf{J} = \{J_{\mathbf{i}}(t) : \mathbf{i} \in \mathcal{I}, t \ge 0\}$, with the set of states $\mathcal{E}$. In a similar way can be modeled each subsystem $\mathbf{i}_k$ of any level k, which components $J_{\mathbf{i}}^{(k)}(t)$ will be considered as binary processes

$$J_{\mathbf{i}}^{(k)}(t) = \begin{cases} 0 & if \quad \mathbf{i}_k\text{-th subsystem of } k\text{-th level works} \\ 1 & if \quad \mathbf{i}_k\text{-th subsystem of } k\text{-th level fails} \end{cases}$$

According to the structure function of the system, the state of its units determines the state of appropriate subsystems and the whole system in sense of its reliability. After the repair of failed units, a subsystem or the whole system, they go to the initial states. Denote by $E_{\mathbf{i}}^{(k)}$ and $\bar{E}_{\mathbf{i}}^{(k)}$ the sets of the working and failure states for $\mathbf{i}$-th subsystem of the k-th level. Then, the working period of the whole system is given by the relation

$$W^{(0)} = \inf\{t : \; J^{(0)}(t) \in \bar{E}^{(0)}\} = \inf\{t : \; J^{(1)}_{(i_1,j)}(t) \in \bar{E}^{(1)}_{(i_1,j)}, \; j = \overline{1, n_1}\}$$

Analogously for each subsystem $\mathbf{i}_k$ of any k-th level one has

$$W_{\mathbf{i}}^{(k)} = \inf\{t : \; t \le W^{(k)}, \; J_{\mathbf{i}}^{(k)}(t) \in \bar{E}_{\mathbf{i}}^{(k)}\} = \inf\left\{t : \; t \le W^{(k)}, \; J^{(k+1)}_{(\mathbf{i}_k,j)}(t) \in \bar{E}^{(k+1)}_{(\mathbf{i}_k,j)}, \; j = \overline{1, n_{\mathbf{i}}^{(k)}}\right\}. \quad (1)$$

Therefore, considering the working period distributions of some subsystem as its life times one can investigate in the same way the reliability function of the subsystem of the next level. However, since the initial information about the system is given for the lowest level only, the problem should be solved starting from the lowest level.

To calculate the c.d.f. of any subsystem $\mathbf{i}_k$ of any level k according to the definition of the working period by formula (1) one has

$$W_{\mathbf{i}}^{(k)}(t) = \mathbf{P}\{W_{\mathbf{i}}^{(k)} \le t\} = 1 - \prod_{\mathbf{j}\in \bar{E}_{\mathbf{i}}^{(k)}} [1 - \pi_{\mathbf{j}}(t)],$$

where $\pi_{\mathbf{j}}(t)$ is the probability distribution for the process $\mathbf{J}^{(k)}$ to be in failure state $\mathbf{j} \in \bar{E}_{\mathbf{i}}^{(k)}$ at a time t. Therefore, the problem is divided into two parts:

- to investigate the process $J_{\mathbf{i}}^{(k)}(t)$ describing behavior of any subsystem of each level;
- to find the working period $W_{\mathbf{i}}^{(k)}$ for any subsystem of each level.

As the problems have identical solution for any subsystem they will be considered in some general construction.

4 The Subsystems Behavior Investigation

For investigation of the separate subsystem with m subsubsystems by using markovization approach we consider a multi-dimensional absorbing (stopped) Markov process $\mathbf{Z} = (\mathbf{J}(t),\, \mathbf{X}(t),\; t < W)$ with general states space $\hat{\mathcal{E}} = \mathcal{E} \times R^m$, where components of the additional vector $\mathbf{X}$ describe the time passed from entering to the appropriate states. Denote by

$$\pi_{\mathbf{j}}(t; \mathbf{x})\mathbf{dx} = \pi_{(j_1,\dots j_m)}(t;\, x_1,\dots x_m)dx_1\dots dx_m = \mathbf{P}\{J_i(t) = j_i,\; X_i(t) \in dx_i,\; i = \overline{1,m},\;\; t < W\} \qquad (2)$$

the probability density function (p.d.f.) of the process' $\mathbf{Z}$ state $(\mathbf{j},\, \mathbf{x})$ at its separate working period W. In order to apply the methods of DSRP denote by:

- $S_1, S_2, \dots S_n, \dots$—the times of jumps of the process $\mathbf{J}$ that coincide with times of failure or renovation of elements of the system (the epochs, in which some of components of the process $\mathbf{Z}$ became equal zero). We will refer to S_n as times of k-th type renovation if $X_k(S_n + 0) = 0$;
- $\mathbf{J}_n = \mathbf{J}(S_n + 0),\;\; \mathbf{X}_n = \mathbf{X}(S_n + 0),\;\; K_n = \#\{k:\; X_k(S_n + 0) = 0\}$
- $\mathbf{j}(k) = (j_1, \dots, \bar{j}_k, \dots j_m),\; \bar{j}_k = 1 - j_k, \quad \mathbf{x}(k) = (x_1, \dots, x_{k-1}, 0, x_{k+1}, \dots x_m),$
- $\mathbf{N} = \{N_{\mathbf{j}}^{(k)}(t;\, \mathbf{dx}):\;\; 1 \le k \le m\}$ embedded renewal process with general renewal states, which component $N_{\mathbf{j}}^{(k)}(t;\, \mathbf{dx})$ is the k-th type embedded renewal process with the set of the renovation states $(\mathbf{j},\, \mathbf{dx})$,
$$N_{\mathbf{j}}^{(k)}(t;\, \mathbf{dx}) = \sum_{n\ge 0} \delta_{k,K_n} 1_{\{[0,t],\mathbf{j},\,\mathbf{dx}\}}(S_n,\, \mathbf{J}_n,\, \mathbf{X}_n);$$
- $H_{\mathbf{j}}^{(k)}(t;\, \mathbf{dx}) = \mathbf{E}N_{\mathbf{j}}^{(k)}(t, \mathbf{dx})$ —k-th type embedded renewal function with the set of renovation states $(\mathbf{j},\, \mathbf{dx})$.

Remark. It is necessary to note that for $\mathbf{j} \in \bar{E}$ the embedded renewal function coincides with the probability of absorbtion by this state, $H_{\mathbf{j}}^{(k)}(t;\, \mathbf{dx}) = \pi_{\mathbf{j}}(t; \mathbf{dx}(k))$.

Denote also by $\Gamma_{j_k}(x)$ the c.d.f. of the duration of stay of the j_k-th component of the process $\mathbf{Z}$ at its state and by $\gamma_{j_k}(x)$ its hazard rate function (h.r.f.),

$$\Gamma_{j_k}(x) = \delta_{j_k,\,0}A_k(x) + \delta_{j_k,\,1}B_k(x), \quad \gamma_{j_k}(x) = \frac{\Gamma'_{j_k}(x)}{1 - \Gamma_{j_k}(x)}.$$

Remind that for p.d.f. $\Gamma(x)$ of any r.v. Γ and appropriate conditional distribution the following representations

$$\Gamma(x) = 1 - \exp\left\{-\int_0^x \gamma(w)dw\right\}; \qquad \mathbf{P}\{\Gamma > y \,|\, \Gamma > x\} = \frac{1-\Gamma(y)}{1-\Gamma(x)} = \exp\left\{-\int_x^y \gamma(w)dw\right\}.$$

hold. In order to give the Laplace transform (LT) $\tilde{\pi}(s;\, \mathbf{v})$ of the p.d.f. $\pi(t;\, \mathbf{x})$

$$\tilde{\pi}_{\mathbf{j}}(s;\, \mathbf{v}) \equiv \int_0^\infty \int_{R^m} e^{-st-\mathbf{v}'\mathbf{y}} \pi_{\mathbf{j}}(t;\, \mathbf{y})\, dt\, \mathbf{dy}$$

let us denote by $v = \sum_{1\le i\le m} v_i$ and $v(k) = \sum_{i\ne k} v_i$ and introduce the following functions

$$\begin{aligned}
\phi_{\mathbf{j}}^{(l)}(s,\, \mathbf{x},\, v) &= \int_0^\infty \exp\left\{-(s+v)t - \sum_{1\le i\le m} \int_{x_i}^{x_i+t} \gamma_{j_i}(\xi)d\xi\right\} \gamma_{j_l}(x_l+t)dt; \\
\psi_{\mathbf{j}}(s,\, \mathbf{x},\, v) &= \int_0^\infty \exp\left\{-(s+v)t - \sum_{1\le i\le m} \int_{x_i}^{x_i+t} \gamma_{j_i}(\xi)d\xi\right\} dt
\end{aligned}$$

that represent LT of the probability density of one-step transition of the process $\mathbf{N}$ from the state $(\mathbf{j},\, \mathbf{x})$ to the state $(\mathbf{j}(l),\, \mathbf{y})$ in the result of l-th type renovation directly before jump, and appropriate LT of the probability density of the duration of stay at the state $\mathbf{j}$. With these notations the following theorem holds

Theorem 1. *The LT $\tilde{\pi}_{\mathbf{j}}(s;\, \mathbf{v})$ of the p.d.f. $\pi_{\mathbf{j}}(t;\, \mathbf{x})$ for $\mathbf{j} \in E$ can be expressed in forms*

$$\begin{aligned}
\tilde{\pi}_{\mathbf{0}}(s;\, \mathbf{v}) &= \psi_{\mathbf{0}}(s;\, \mathbf{0},\, v) + \sum_{1\le k\le m} \int_0^\infty \int_{R^{m-1}} e^{-su-\mathbf{v}'\mathbf{x}(k)} H_{\mathbf{0}}^{(k)}(du;\, \mathbf{dx}) \psi_{\mathbf{0}}(s;\, \mathbf{x},\, v); \\
\tilde{\pi}_{\mathbf{j}}(s;\, \mathbf{v}) &= \sum_{1\le k\le m} \int_0^\infty \int_{R^{m-1}} e^{-su-\mathbf{v}'\mathbf{x}(k)} H_{\mathbf{j}}^{(k)}(du;\, \mathbf{dx}) \psi_{\mathbf{j}}(s;\, \mathbf{x},\, v). \blacksquare \qquad (3)
\end{aligned}$$

In order to get the LT

$$\tilde{h}_{\mathbf{j}}^{(l)}(s;\, \mathbf{dy}) \equiv \int_0^\infty e^{-st} H_{\mathbf{j}}^{(l)}(dt; \mathbf{dy}) \quad \text{and} \quad h_{\mathbf{j}}^{(l)}(s;\, \mathbf{v}) \equiv \int_0^\infty \int_{R^{m-1}} e^{-st-\mathbf{v}'\mathbf{y}(l)} H_{\mathbf{j}}^{(l)}(dt; \mathbf{dy})$$

of the embedded renewal functions $H_{\mathbf{j}}^{(l)}(dt;\, \mathbf{dy})$ remark that in the definition of the renewal functions of the one-step transition probability density from the state $(\mathbf{j},\, \mathbf{x})$ to the state $(\mathbf{j}(l),\, \mathbf{y})$ directly after jump is used. Taking into account this remark the following theorem can be proved

Theorem 2. *The LT $h_{\mathbf{j}}^{(l)}(s;\, \mathbf{v})$ of the embedded renewal functions $H_{\mathbf{j}}^{(l)}(dt; \mathbf{dy})$ for $\mathbf{j} \in E$ such that $\mathbf{j}(l) \in E$ are*

$$\begin{aligned}
h_{\mathbf{e}_l}^{(l)}(s;\, \mathbf{v}) &= \phi_{\mathbf{0}}(s;\, \mathbf{0},\, v(l)) + \sum_{1\le k\le m} \int_{R^{m-1}} \tilde{h}_{\mathbf{0}}^{(k)}(s;\, \mathbf{dx}) e^{-\mathbf{v}'\mathbf{x}} \phi_{\mathbf{0}}^{(l)}(s;\, \mathbf{x},\, v(l)) \\
h_{\mathbf{j}}^{(l)}(s;\, \mathbf{v}) &= \sum_{1\le k\le m} \int_{R^{m-1}} \tilde{h}_{\mathbf{j}}^{(k)}(s;\, \mathbf{dx}) e^{-\mathbf{v}'\mathbf{x}} \phi_{\mathbf{j}}^{(l)}(s;\, \mathbf{x},\, v(l)) \qquad (4)
\end{aligned}$$

and for $\mathbf{j}(l) \in \bar{E}$ such that $\mathbf{j}(l) \in E$ are

$$h_{\mathbf{j}}^{(l)}(s;\, \mathbf{v}) = \sum_{1\le k\le m} \int_{R^{m-1}} \tilde{h}_{\mathbf{j}}^{(k)}(s;\, \mathbf{dx}) \phi_{\mathbf{j}}^{(l)}(s;\, \mathbf{x},\, v(l)). \blacksquare \qquad (5)$$

The results above show that to investigate the problem in general case the solution of complex integral equations is needed. On the other hand they show that the problem could be reduced to the investigation of the functions $\psi_{\mathbf{j}}(.;.,.)$ and $\phi_{\mathbf{j}}(.;.,.)$.

5 Exponential case. An Example

Note that under exponential distributions of the life and repair times the functions $\phi_{\mathbf{j}}^{(l)}(s;\,\mathbf{x},\,v)$ and $\psi_{\mathbf{j}}(s;\,\mathbf{x},\,v)$ do not depend on the additional variables $\mathbf{x}$,

$$\phi_{\mathbf{j}}^{(l)}(s;\,\mathbf{x},\,v) = \frac{\gamma_{j_l}}{s+v+\gamma_{\mathbf{j}(l)}}, \qquad \psi_{\mathbf{j}}(s;\,\mathbf{x},\,v) = \frac{1}{s+v+\gamma_{\mathbf{j}}}.$$

This remark makes it possible to simplify the general equations and calculate the LT of the process $\mathbf{J}$ state probabilities $\tilde{\pi}_{\mathbf{j}}^{(W)}(s) = \tilde{\pi}_{\mathbf{j}}^{(W)}(s;\,\mathbf{0})$.

Here we represent the appropriate results for the simplest example with only two elements in a system. Denote: $\mathbf{0} = (0,0),\ \mathbf{1} = (0,1),\ \mathbf{2} = (1,0),\ \mathbf{3} = (1,1),\ .$ With these notations one has

$$\begin{aligned}
\tilde{\pi}_{\mathbf{0}}(s) &= \frac{1}{s+\gamma_{\mathbf{0}}} + h_{\mathbf{0}}^{(1)}(s)\frac{1}{s+\gamma_{\mathbf{0}}} + h_{\mathbf{0}}^{(2)}(s)\frac{1}{s+\gamma_{\mathbf{0}}};\\
\tilde{\pi}_{\mathbf{k}}(s) &= h_{\mathbf{k}}^{(k)}(s)\frac{1}{s+\gamma_{\mathbf{k}}} \qquad (k=1,2);\\
\tilde{\pi}_{\mathbf{3}}(s) &= h_{\mathbf{1}}^{(1)}(s)\frac{\alpha_2}{s+\gamma_{\mathbf{1}}} + h_{\mathbf{2}}^{(2)}(s)\frac{\alpha_1}{s+\gamma_{\mathbf{2}}},
\end{aligned}$$

where the functions $h_{\mathbf{j}}^{(k)}(s)$ satisfy to the equations

$$\begin{aligned}
h_{\mathbf{0}}^{(k)}(s) &= h_{\mathbf{k}}^{(k)}(s)\frac{\beta_k}{s+\gamma_{\mathbf{k}}} \qquad (k=1,2);\\
h_{\mathbf{k}}^{(k)}(s) &= \frac{\alpha_k}{s+\gamma_{\mathbf{0}}}(1+h_{\mathbf{0}}^{(1)} + h_{\mathbf{0}}^{(2)}(s)) \qquad (k=1,2).
\end{aligned}$$

The solutions of these equations are (for $\bar{\mathbf{k}} = (\bar{k}_1,\,\bar{k}_2)$)

$$\begin{aligned}
h_{\mathbf{0}}^{(k)}(s) &= \frac{\beta_k(s+\gamma_{\bar{\mathbf{k}}})}{(s+\gamma_{\mathbf{0}})(s+\gamma_{\mathbf{1}})(s+\gamma_{\mathbf{2}}) - \alpha_1\beta_1(s+\gamma_{\mathbf{2}}) - \alpha_2\beta_2(s+\gamma_{\mathbf{1}})} \qquad (k=1,2)\\
h_{\mathbf{k}}^{(k)}(s) &= \frac{\alpha_k(s+\gamma_{\mathbf{1}})(s+\gamma_{\mathbf{2}})}{(s+\gamma_{\mathbf{0}})(s+\gamma_{\mathbf{1}})(s+\gamma_{\mathbf{2}}) - \alpha_1\beta_1(s+\gamma_{\mathbf{2}}) - \alpha_2\beta_2(s+\gamma_{\mathbf{1}})} \qquad (k=1,2)
\end{aligned}$$

Therefore for working period LST $\tilde{w}(s) = \int_0^\infty e^{-st}W(dt) = \tilde{\pi}_{\mathbf{3}}(s)$ one has

$$\tilde{\pi}_{\mathbf{3}}(s) = \frac{\alpha_1\alpha_2(2s+\gamma_{\mathbf{1}}+\gamma_{\mathbf{2}})}{(s+\gamma_{\mathbf{0}})(s+\gamma_{\mathbf{1}})(s+\gamma_{\mathbf{2}}) - \alpha_1\beta_1(s+\gamma_{\mathbf{2}}) - \alpha_2\beta_2(s+\gamma_{\mathbf{1}})},$$

that coincides with the results obtained by classic Markov approach.

References

[1] B. Dimitrov, V. Rykov, P. Stanchev (2002) On Multi-State Reliability Systems. In: *Proceedings MR-2002.* Trondheim (Norway) June 17-21, 2002.

[2] V. Rykov, B. Dimitrov, D. Green Jr., P. Snanchev (2004) Reliability of complex hierarchical systems with fault tolerance units. In *Proceedings MMR-2004.* Santa Fe (U.S.A.) June, 2004.)

[3] Yu. K. Belyaev. Line Markov processes and their application to problems of the theory of reliability. (English) Select. Translat. Math. Statist. Probab. 9, 233-250 (1971); translation from Trudy VI vsesojuzn. Sovescan. po Teor. Verojatn. Mat. Statist. Vil'njus-Palanga 1960, 309-323 (1962).

[4] Н.П. Бусленко. К теории сложных систем. *Изв. АН ССР . Техн. киберн.*, М.: № 5, 1963

[5] Н.П. Бусленко. Об одном классе сложных систем. *Проблемы прикладной математики и механики*, М.: "Наука", 1971

[6] Н.П. Бусленко, В.В. Калашников, И.Н. Коваленко. *Лекции по теории сложных систем.* М.: "Советское радио", 1973, 440 с.

[7] В.В. Калашников. *Topics on Regenerative Processes*, CRC Press, Boca Raton, 1994.

[8] И.Н. Коваленко. О некоторых классах сложных систем. *Изв. АН ССР . Техн. киберн.*, М.: № 6, 1964, №№ 1, 3, 1965.

[9] И.Н. Коваленко. *Исследования по анализу надежности сложных систем.* Киев: "Наукова думка", 1976, 212с.

[10] В.В. Рыков, М.А. Ястребенецкий. О регенерирующих процессах с нечколькими типами точек регенерации. *Кибернетика.* № 3. С.82-86. Киев. 1971.

[11] В.В. Рыков. Регенерирующие процессы с вложенными периодами регенерации и их применение при исследовании приоритетных систем массового обслуживания. *Кибернетика.* N 6 Киев. 1975.

[12] В.В. Рыков. Исследование одноканальной системы общего вида методом регенерирующих процессов. I. *Изв. АНСССР. Технич. киберн.* 1983. N.6. С.13-20.

[13] В.В. Рыков. Исследование одноканальной системы общего вида методом регенерирующих процессов. II. Исследование основных процессов на периоде регенерации. *Изв. АНСССР. Технич. киберн.* 1984. N.1. С.126-132.

[14] В.В. Рыков. Два подхода к декомпозиции сложных иерархических систем. Агрегативные системы. *Автоматика и телемеханика*, № 12, 1997, сс. 140-149.

[15] V.V. Rykov, S.Yu. Jolkoff. Generalized regeneranive processes with embedded regeneranion periods and their applications. *MOS. Ser. Optimization.* V.12. (1981). N.4. P.575-591.