Methods of Asymptotic Analysis for Nonlinearly Perturbed Processes and Systems

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Abstract

The lecture presents a new book, Cyllenberg and Silvestrov (2008), written by the author in co-operation with Professor Mats Gyllenberg (University of Helsinki). The book is devoted to studies of quasi-stationary phenomena in nonlinearly perturbed stochastic systems. The core of the quasi-stationary phenomenon is that one can observe something that resembles a stationary behaviour of the system before the lifetime goes to the end. Examples of stochastic systems, in which quasi-stationary phenomena can be observed, are various queuing systems and reliability models, in which the lifetime is usually considered to be the time in which some kind of a fatal failure occurs in the system. Another class of examples of such stochastic systems is supplied by population dynamics or epidemic models. In population dynamics models, the lifetimes are usually the extinction times for the corresponding populations. In epidemic models, the role of the lifetime is played by the time of extinction of the epidemic in the population. A ruin for risk processes can be also considered as an example of such phenomenon. An important characteristic feature of these models is a non-linear character of perturbation that essentially complicates the asymptotic analysis of quasi-stationary phenomena. New methods of asymptotic analysis for nonlinearly perturbed processes and systems developed in the book are based on the exponential asymptotic expansions for nonlinearly perturbed renewal equations. These methods are applied to nonlinearly perturbed regenerative processes, semi-Markov processes, and continuous time Markov chains with absorption. The asymptotic results include: mixed ergodic theorems (for the state of the process) and limit theorems (for the lifetimes) that describe transition phenomena; mixed ergodic and large deviation theorems that describe pseudo- and quasi-stationary phenomena; exponential expansions in mixed ergodic and large deviation theorems; theorems on convergence of quasi-stationary distributions; and asymptotic expansions for quasi-stationary distributions. Applications to the analysis of quasi-stationary phenomena in various models of nonlinearly perturbed stochastic systems considered in the book pertain to models of highly reliable queuing systems, M/G queuing systems with quick service, stochastic systems of birth-death type, including epidemic and population dynamics models, metapopulation dynamic models, and perturbed risk processes. The book contains an extended bibliography of works in the area. New directions for the research in the area are commented in the book and they will be also discussed in the lecture.

References

Cyllenberg, M. and D. Silvestrov (2008). *Quasi-stationary Phenomena in Nonlinearly Perturbed Stochastic Systems*, Volume 44 of *De Gruyter Expositions in Mathematics*. Berlin: Walter de Gruyter.