On an identification of a distribution by Birnbaum-Saunders data

I.N. Volodin
Depart. of Math Stat
Kazan University
Kremlyevskaya st. 18, Kazan
Russia
Igor.Volodin@ksu.ru

M.I. Muflihanov
Depart. of Math Stat
Kazan University
Kremlyevskaya st. 18, Kazan
Russia
phoenix@mi.ru

A classical probability model of a fatigue crack development under a periodic loading could lead to two different distributions of the break down time in accordance with the choice of an approximation method of sums of independent random variables. We provide arguments that the data presented in Birnbaum-Saunders (1962) represent more a sample from the inverse Gaussian distribution, than from the author's distribution.

Both distributions follow from the identical assumptions for a probability model construction, where the size X_k of the crack after the k-th loading of an object with thickness a is defined by the value of X_{k-1} through the relationship

$$X_k = X_{k-1} + \xi_k\, g(X_{k-1}),\ k = 1,\, 2, \ldots,\ X_0 > 0, \tag{1}$$

where $\{\xi_k,\ k = 1,\, 2, \ldots\}$ is a sequence of independent identically distributed random variables with the finite expectation μ and variance σ^2. A positive and continuous function $g(\,\cdot\,)$ defines a physics of break down.

Under the assumption that an increment for each particular loading is sufficiently small and the break down appears after a significant amount of loadings, it follows from (1) that the at the time of break down τ the equality

$$h(a) = \int_{X_0}^{X_n} \frac{dx}{g(x)} \approx \sum_{k=1}^{\tau} \xi_k$$

takes place. Hence, the mean time before the break down is $E\,\tau = h(a)/\mu$ and this value should be the same for the Birnbaum-Saunders and Inverse Gaussian distributions.

Actually the mean values are different. A detailed statistical analysis of three samples presented in the paper Birnbaum and Saunders (1962) is provided. It follows that these distributions can be equally considered on the life-time distribution. Our conclusion for a preference of IG-distribution for BS-data is based primarily on the difference in formulae of mean values of the break down time. Most probably, data for less resistant for a fatigue cracks objects, that correspond to small values of parameter α, may clarify the difference in psychical and probabilistic nature of these distributions.

Reference

[1] Birnbaum, S.Z.W. Saunders,S.C. (1962). A new family of life-distribution, *J. Appl. Probab., 6,* pp. 319-327.