Asymptotic approximation of inverse moments of nonnegative random variables

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Abstract

Let $\{Z_n,\ n \geq 1\}$ be a sequence of independent nonnegative r.v.'s (random variables) with finite second moments. it is shown that under a Lindeberg-type condition, the α-th inverse moment $E\{a + X_n\}^{-\alpha}$ can be asymptotically approximated by the inverse of the α-th moment $\{a + EX_n\}^{-\alpha}$ where $a > 0,\ \alpha > 0$, and $\{X_n\}$ are the naturally-scaled partial sums. Furthermore, it is shown that, when $\{Z_n\}$ only possess finite r-th moments, $1 \leq r < 2$, the preceding asymptotic approximation can still be valid by using different norming constants which are the standard deviations of partial sums of suitably truncated $\{Z_n\}$. The inverse moments appear in many practical applications. For example, they may be applied in Stein estimation and post-stratification, evaluating risks of estimators and powers of tests. In addition, they appear in certain problems of reliability and life testing, insurance and financial mathematics, complex systems, and others.