Logic Algebra in reliability analysis of Multi-State System

Elena Zaitseva
Dep. of Informatics
University of Zilina,
ul.Univerzitna 8215/1, 01026
Zilina, Slovakia
Elena.Zaitseva@fri.uniza.sk

Abstract

The reliability of Multi-State System (MSS) is analysed in this paper. In a MSS, both the system and its components may experience more than two reliability states. Many practical and theoretical problems needs still to be solved in this area. One of the crucial ones is to identify how a change in a state of an individual component or changes in states of several ones affect(s) the system reliability. The Multiple-Valued Logic (MVL) tools are employed for handling this problem in this paper. In the paper the structure function and Logical Differential Calculus of MVL function are combined to evaluate the dynamic behaviour of a MSS. The Logical Differential Calculus extends potentialities of structure function tool to analyse also the MSS dynamical properties. The evaluation of MSS components changes is considered in this paper.

1 Introduction

Reliability evaluation approaches exploit a variety of tools for system modeling and reliability indices calculation. A discrete model has been used in reliability analysis frequently and well known two mathematical tapes of this model: Binary System and *Multi-State System* (MSS).

In a Binary System, the system and its components are allowed to have only two possible states (completely failed and perfect functioning). But this approach fails to describe many situations where the system can have more than two distinct states (Lisnianski and Levitin 2003, Pham 2003, Xie and Dai and Poh 2004). In a MSS, both the system and its components may experience more than two states, for example completely failed, partially failed, partially functioning and perfect functioning. A MSS reliability analysis is a more flexible approach to evaluate system reliability.

Reliability indices of a MSS have been computed as the reliability of the specified level of the system states or its modification depending on the change of the system component states (Shooman 2002,Ushakov 1994). These indices evaluate of relative importance of the various MSS components on the system reliability. As a rule the methods for evaluation of MSS components importance are applied system boundary states (they are system states for which change a system component state causes change in level of system functioning). There are different mathematical tools to get these indices (Lisnianski and Levitin 2003, Xie and Dai and Poh 2004). One of them is structure function tool.

In papers (Zaitseva 2003, Zaitseva and Levashenko 2006) have been considered application of the mathematical tools of *Multiple-Valued Logic* (MVL) in MSS reliability analysis. The MSS structure function is interpreted as MVL function in these papers. There are many mathematical approaches and tools in MVL for analysis and estimation of MVL functions. In particular the Logical Differential

Calculus allows to analysis dynamic properties of MVL function: this approach determine changes in MVL function depending of changes of its variables. One method of Logical Differential Calculus is used for discover boundary states of MVL function and allows to determine influence change of every variable value to change of MVL function value. This method is named Direct Partial Logic Derivative and can be used for determination of boundary states of a MSS by the structure function of this system.

Application of Direct Partial Logic Derivative for evaluation of the MSS reliability has been considered in (Zaitseva and Levashenko 2006, Zaitseva and Puuronen 2008). Based on this method new means for estimation of relative importance of the system component on MSS have been obtained. Dynamic Reliability Indices (DRIs) are computed by theis evaluation. DRIs characterize the change of the MSS reliability that is caused by the change of a component state.

In this paper the method of MVL for estimation of MSS reliability is presented. The formalization of MSS in terms of MVL is considered. Methods of Logical Differential Calculus are used to evaluate changes of the MSS reliability (MSS states) depending of changes of the system component or some system components.

2 MVL mathematical model of MSS

The MSS structure function permits to define the system reliability (system state) depending on its components states. In terms of MVL the structure function of MSS with n component and m states of reliability for system and its components is described as:

$$\phi(x_1, \ldots, x_n) = \phi(\boldsymbol{x})\colon \{0, \ldots, m\text{-}1\}^n \to \{0, \ldots, m\text{-}1\}. \tag{1}$$

where x_i is the i-th variable of MVL function and state of the i-th component: $x_i = 0, \ldots, m$-1; n is number of variable of MVL and number of the MSS components; m is number of values for MVL function and its variables and it is number of reliability level for system and ots component: from complete failure (it is 0) to the perfect functioning (it is m-1).

The interpretation of the structure function (1) as MVL function permits to apply MVL tools for reliability analysis of the MSS (Zaitseva 2003). Direct Partial Logic Derivative method describes dependence between changes of system component states and MSS reliability.

A Direct Partial Logic Derivatives $\partial\phi(j\to h)/\partial x_i(a\to b)$ of a MSS structure function $\phi(\boldsymbol{x})$ with respect to variable x_i reflects the fact of changing of system reliability from j to h when the component state x_i is changing from a to b (Zaitseva 2003):

$$\partial\phi(j\to h)/\partial x_i(a\to b) = \begin{cases} m-1, & \text{if } \phi(a_i, \boldsymbol{x}) = j \,\&\, \phi(b_i, \boldsymbol{x}) = h \\ 0, & \text{in the other case} \end{cases} \tag{2}$$

where $\phi(a_i, \boldsymbol{x}) = \phi(x_1,\ldots, x_{i-1}, a, x_{i+1},\ldots, x_n)$ and $\phi(b_i, \boldsymbol{x}) = \phi(x_1,\ldots, x_{i-1}, b, x_{i+1},\ldots, x_n)$; $a, b \in \{0, \ldots, m\text{-}1\}$.

Direct Partial Logic Derivative with respect to variables vector for a MSS structure function permits to analyse the system reliability change from j to h when every variable values of this vector changes from a to b (Zaitseva and Levashenko 2007):

$$\partial \phi(j \rightarrow h)/\partial \boldsymbol{x}^{(p)}(\boldsymbol{a}^{(p)} \rightarrow \boldsymbol{b}^{(p)}) = \begin{cases} m-1, \text{ if } \phi(a_{i_1},\ldots,a_{i_p},\boldsymbol{x}) = j \,\&\, \phi(b_{i_1},\ldots,b_{i_p},\boldsymbol{x}) = h \\ 0, \quad \text{in the other case} \end{cases}. \tag{3}$$

where $\phi(\boldsymbol{a}^{(p)}, \boldsymbol{x}) = \phi(a_{i_1},\ldots,a_{i_p},\boldsymbol{x})$ is value of structure function, when $x_{i_1} = a_{i_1},\ldots,x_{i_p} = a_{i_p}$ and $\phi(\boldsymbol{b}^{(p)}, \boldsymbol{x}) = \phi(b_{i_1},\ldots,b_{i_p},\boldsymbol{x})$ is value of structure function, when $x_{i_1} = b_{i_1},\ldots,x_{i_p} = b_{i_p}$.

Some assumption will be used in the MSS mathematical model for evaluation of important relevant component on the system. They assume that the MSS structure function (*a*) the structure function is monotone and $\phi(\boldsymbol{s})=s$ ($s\in\{0, \ldots, m-1\}$); (*b*) all components are *s*-independent and are relevant to the system.

MSS failure measures in reliability analysis plays significant role. In Direct Partial Logic Derivative terminology the MSS failure is represented as the changing of the function value $\phi(\boldsymbol{x})$ from *j* into zero and as decrease of a system components availability vector $\boldsymbol{x}^{(p)}$ from $\boldsymbol{a}^{(p)}$ to $\boldsymbol{b}^{(p)}$. Because the structure function is monotone (assumption (*a*)) a MSS failure is declared by a change function $\phi(\boldsymbol{x})$ from "1" into zero and decreases of every of *p* system components availability from a_{i_j} to $(a_{i_j}-1)$:

$$\partial \phi(1 \rightarrow 0)/\partial \boldsymbol{x}^{(p)}(\boldsymbol{a}^{(p)} \rightarrow \tilde{\boldsymbol{a}}^{(p)}), \tag{4}$$

where $\tilde{\boldsymbol{a}}^{(p)} = (\tilde{a}_{i_1},\ldots,\tilde{a}_{i_p}) = ((a_{i_1}-1),\ldots,(a_{i_p}-1))$ and $a_{i_j} \in \{1,\ldots,(m-1)\}$.

MSS repair is possible only for reparable system (Pham 2003, Xie and Dai and Poh 2004). A MSS repair can be considered for replacement of failed components (components state change from zero into (*m*-1)) and for renewal of failed components (components state change from zero into some level $b\in\{1, \ldots, m-1\}$). The first case is more important for real-world systems and it is considered in more detail below.

The MSS repair in Direct Partial Logic Derivative terminology is declared as the structure function change from zero into *h* ($\phi(\boldsymbol{x})$: $0 \rightarrow h$) and as *p* failed system components changes from zero into (*m*-1):

$$\partial \phi(0 \rightarrow h)/\partial \boldsymbol{x}^{(p)}(\boldsymbol{0} \rightarrow (\boldsymbol{m}\text{-}\boldsymbol{1})), \tag{5}$$

where $h\in\{1, \ldots, m-1\}$; $\boldsymbol{0} = (\underbrace{0,\ldots,0}_{p})$ and $(\boldsymbol{m}-\boldsymbol{1}) = (\underbrace{(m-1),\ldots,(m-1)}_{p})$.

Equations (4) and (5) allow to describe the MSS behaviour for system failure and system repair. But the mathematical model (2) and (3) can be used for analysis all changes of the MSS reliability caused by changes of system component states.

3 Measures of the MSS

There are two groups of DRIs: *Component Dynamic Reliability Indices* (CDRIs) and *Dynamic Integrated Reliability Indices* (DIRIs). CDRIs allows measuring an influence of each individual component to the system reliability. DIRIs represent how change of one of system components impacts to the system reliability. Calculation of these induces have been considered in (Zaitseva and Levashenko 2006, Zaitseva and Levashenko 2007, Zaitseva and Puuronen 2008).

But application of MVL allows to determine one more group of the MSS measures. It is *Reliability Function* (RF) (Lisnianski and Levitin 2003). The MSS RF is calculated based the structure function presentation by (1). Calculation of probabilities of every level of the MSS reliability by RF uses two steps. A determination of the MSS boundary states is implemented on the first step. Boundary states are computed by Direct Partial Logic Derivatives (2) and (3). The MSS structure function is transformed into probabilistic description for every level of the system reliability by orthogonalization on the second step.

Therefore two types of the measures are considered as DRIs and RF. The mathematical tools of MVL are used for calculation of these measures.

Acknowledgements

This work was partially supported by grants of Vega 1/0664/08.

References

Lisnianski, A. Levitin, G. (2003). Multi-state system reliability. Assessment, Optimization and Applications. World Scientific.

Pham, H. (2003). Handbook of reliability engineering. London: Springer Verlag.

Shooman, M.L. (2002). Reliability of computer systems and networks: Fault tolerance, Analysis, and Design. New York: John Wiley & Sons, Inc.

Ushakov, I. (1994). Handbook of reliability engineering. New York:Wiley.

Xie, M. & Dai, Y.-S. & Poh, K.-L. (2004). Computing System Reliability. Models and Analysis**.** New York: Kluwer Academic Publishers.

Zaitseva, E.N. (2003). Reliability Analysis of Multi-State System, *Dynamical Systems and Geometric Theories* 1(2), pp.213-222.

Zaitseva, E. Levashenko, V. (2006). Dynamic Reliability Indices for Parallel, Series and *k*-out-of-*n* Multi-State Systems. In *Proc. of the IEEE 52th Annual Reliability and Maintainability Symposium (RAMS)*, pp.253 – 259. Newport Beach, USA.

Zaitseva, E., Levashenko, V. (2007). Investigation Multi-State System Reliability by Structure Function. In *Proc. of the Int. Conf. on Dependability of Computer System (DepCoS-RELCOMEX'07)*, *pp.81-88*, Szklarska Poreba, Poland.

Zaitseva, E., Puuronen, S. (2008). Representation and Estimation of Multi-State System Reliability by Decision Diagrams. In *Safety, Reliability and Risk Analysis: Theory, Methods and Application. S.Martorell, C.G.Soares, Barnett J. (Eds), pp.1995-2002*. Taylor & Francis Group, London,.