MARKOVIAN AND SEMI-MARKOVIAN CLOSED QUEUING SYSTEMS WITH TWO TYPES OF SERVICE AS MATHEMATICAL MODELS OF RELIABILITY AND MAINTENANCE

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We discuss new Markovian and semi-Markovian models of reliability and maintenance of recoverable complex standby systems. In these models the duration of replacement (switch-over) of failed main element by standby one is taken into consideration [1]–[9].

The necessity for the consideration of this factor has long been emphasized by leading reliability theory and practice experts [10]–[12].

The fact is that in the absolutely most cases of investigations on reliability, the replacement of the failed element is not considered as separate, independent operation.

In comparatively simple systems there simply is no such necessity, as this operation may be included into more complex operation of restoration (renewal) meaning the sequence of two operations: repair and replacement.

However, in complex, multi-element recoverable standby systems after failure of some main element there appears in the first place the necessity of its replacement with serviceable standby one, so that here replacement is quite naturally distinguished as an independent operation of service. In other cases it is supposed that replacement duration is considerably small compared to the length of restoration (renewal) and it can be neglected (instant replacement).

But in many systems, particularly in complex production ones, replacement operation frequently has the duration of the same order that the duration of repair and even exceeding it is not rare. Therefore, the assumption about instant replacement is quite rough. Such assumption is especially inadmissible in conditions of generalized interpretation of the notions of the theory of reliability given below.

Namely, the failure of some element is understood as the occurrence of such event when this element cannot execute the definite category tasks with stated priority. This may be caused not only by the loss of element serviceability but by other factors as well including its switching over for execution of higher priority tasks, readjustment, "heating", *etc. Other causes of failures may also be named in generalized sense. Within such approach under the renewal time we mean the length of time during which the element is unserviceable in generalized sense, i.e. is not able to serve the mentioned task flow. From the generalized interpretation of the notion of failure and renewal we easily come to the notion of standby in generalized sense, etc.*

The basic model in the form of a closed queuing system with two types of service operations is described in the following way.

The technical system consists of $\boldsymbol{m}$ main and $\boldsymbol{n}$ standby elements. All elements are identical. It is supposed that for normal operation of the system, the serviceability of all $\boldsymbol{m}$ main elements is desired. However, if their number is less than $\boldsymbol{m}$ than the system continues to function but with decreased economical effectiveness. The main elements fail with intensity (and the standby ones with intensity β. The failed main element is replaced with a serviceable standby element if there is such possibility in the system. In the opposite case (all standby elements are non-serviceable, or serviceable standby elements are already intended for the replacement of earlier failed main elements) the replacement of the main element will be done after such possibility arises. The failed elements, main as well as standby ones, are repaired and become identical to the new ones. There are k and l replacement and repair units in the system. The duration of replacement and renewal operations have general distribution functions, $F(t)$ and $G(t)$, respectively.

At present, only some particular cases of the described system are investigated in the reliability theory and queuing theory. Namely:

$\boldsymbol{m} = 1$, $\boldsymbol{n} = 1$; 2) $\boldsymbol{m} = 1$, $\boldsymbol{n} = 2$; 3) $\boldsymbol{m} = 2$, $\boldsymbol{n} = 1$; 4) $M/M/N$, i.e. the repair duration has an exponential distribution, while the replacement duration equals zero (instant replacement); 5) some similar formulations are also investigated.

During the last 5–6 years prof. Kakubavas workgroup succeeded to considerably advance in the indicated direction. In particular, the corresponding models are constructed and partially investigated for the following cases:

1) $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{k}$, $\boldsymbol{l}$ are arbitrary; functions $F(t)$ and $G(t)$ are exponential;

2) $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{l}$ and function $F(t)$ are arbitrary; $\boldsymbol{k} = 1$ and function $G(t)$ is exponential;

3) $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{k}$, $\boldsymbol{l}$ and function $G(t)$ are arbitrary; $\boldsymbol{l} = 1$ and function $F(t)$ is exponential;

4) $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{k}$, $\boldsymbol{l}$ and functions $F(t)$ and $G(t)$ are arbitrary; $\boldsymbol{k} = \boldsymbol{l} = 1$.

In the first case mathematical model represents a system of usual linear differential first order equations (Kolmogorov equations), which in steady mode transforms into the system of linear algebraic equations. The difficulties of its solution are of purely calculating character.

As to the second, third and fourth cases, the corresponding mathematical models are obtained in the form of non-classical boundary-value problems of mathematical physics. Boundary conditions in these problems are nonlocal and represent the system of recursive integral equations. The correct statement of these problems and their further investigation with analytical and numerical methods is a complicated and interesting problem from the point of view of the applied mathematics, as well as from the points of view of the reliability theory and the queuing theory. We got impressive results in this direction.

Then we introduce the function for economic analysis.

This function expresses the profit of examined systems per time unit, taking into account the following values: $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ - profits per time unit from one main and one serviceability standby elements respectively; $\boldsymbol{c}_1$, $\boldsymbol{c}_2$ and $\boldsymbol{c}_3$ - expenditures per time unit on one main, one serviceability standby and one failure elements; $\boldsymbol{c}_4$, $\boldsymbol{c}_5$, $\boldsymbol{c}_6$ and $\boldsymbol{c}_7$ - expenditures per time unit on one working replacement unit, one working renewal (repair) unit, one nonworking replacement unit and one nonworking renewal unit.

Initial characteristics of considered systems - $\boldsymbol{m}$, $\boldsymbol{n}$, $\boldsymbol{k}$, $\boldsymbol{l}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_7$ enter into the expression of profit function via $\boldsymbol{P}(\boldsymbol{i}, \boldsymbol{j})$ – probabilistic characteristics of considered systems. Therefore this function is as following $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{m}, \boldsymbol{n}, \boldsymbol{k}, \boldsymbol{l}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_7)$.

The problems of obtaining of $\boldsymbol{P}(\boldsymbol{i}, \boldsymbol{j})$ characteristics are considered in [1]–[9].

Eventually, the problem of system optimization is stated as problem of mathematical programming (integer programming).

Namely, with those fixed $\boldsymbol{m}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, $\boldsymbol{c}_1, \ldots, \boldsymbol{c}_7$ for the considered system to select such values of the parameters $\boldsymbol{n}$, $\boldsymbol{k}$, $\boldsymbol{l}$ (optimal numbers of standby elements, replacement units and renewal units) so that the profit function $\boldsymbol{F}$ would accept maximum value and to determine this value.

That means the solving of problem of analytical synthesis of multi-element recoverable standby system by economical criterion. We believe, this is the significantly important result in the field of reliability (dependability) theory as well as in the field of structural control of complex systems.

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