A reliability model for safety system-protected object complex with time redundancy.

Pereguda A.I.
Obninsk State Technical University
for Nuclear Power Engineering
Russia
pereguda@iate.obninsk.ru

Abstract

In this paper an advanced safety system-protected object complex reliability model has been proposed, assuming time redundancy caused by protected object inertia. The model can be used to estimate such reliability indices as mean time to failure and probability of failure prior to time t.

1 Introduction

Systems with time redundancy are common in the engineering practice. There are some methods available for the estimation of reliability indices of such systems (Gnedenko and Ushakov 1995). But there is a lack of reliability models for safety system-protected object complex with time redundancy caused by protected object inertia. In the present study we set out to analyze the reliability of such system. We follow Pereguda (2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. Our objective is to provide an asymptotic estimation for such reliability indices as mean time to failure and probability of failure prior to time t.

2 Problem formulation

Consider a safety system-protected object complex. Let safety system and protected object are repairable. Let safety system and protected object are restored to an as-good-as-new state. It is assumed that safety system failures can be detected only during preventive maintenance. All failures are supposed to be independent. By χ_1 denote the time to the first protected object failure. By χ_i, $i = 2, 3, \ldots$ denote the time to the protected object failure after the $(i-1)$-th repair. Let χ_i, $i = 1, 2, \ldots$ be independent and identically distributed (i.i.d.) random variables. By γ_i denote the time to the i-th repair of the protected object. Let γ_i, $i = 1, 2, \ldots$ be i.i.d. random variables. Suppose γ_i, $i = 1, 2, \ldots$ are renewal points of the operation process of the complex. By ξ_1 denote the time to the first failure of the safety system. By ξ_i, $i = 2, 3, \ldots$ denote the time to the failure of the safety system after the $(i-1)$-th repair. Let ξ_i, $i = 1, 2, \ldots$ be i.i.d. random variables. By η_i denote the time to the i-th repair of the safety system. Let η_i, $i = 1, 2, \ldots$ be i.i.d. random variables. Suppose η_i, $i = 1, 2, \ldots$ are renewal points of the operation process of the safety system. Let the preventive maintenance of the safety system is performed at periodic time intervals T. Let the duration of the preventive maintenance of the safety system is θ. By U_n denote the moment of the n-th failure of the safety system. By V_n denote the moment of the n-th repair of the safety system. Then the safety system-protected object complex fails when

$$U_n \le \chi < V_n - \alpha,$$
$$\alpha \le V_n - U_n$$

or when

$$V_{n-1} + T \le \chi < V_{n-1} + (T + \theta) - \alpha;$$
$$V_{n-1} + (T + \theta) + T \le \chi < V_{n-1} + 2(T + \theta) - \alpha;$$
$$\ldots$$
$$V_{n-1} + \left(\left[\frac{\xi_n}{T+\theta}\right] - 1\right)(T + \theta) + T \le \chi < V_{n-1} + \left[\frac{\xi_n}{T+\theta}\right](T + \theta) - \alpha;$$
$$\alpha < \theta$$

where α is an excess time and $[x]$ is an integer part of x. It is assumed that α is a random variable. By ω denote the time to the first failure of the safety system-protected object complex. Our aim is to estimate mean time to failure $E[\omega]$ and $F_\omega(t) = Pr\{\omega \le t\}$.

3 Main results

Since the operation process of the safety system is an alternating renewal process, it follows that

$$U_n = \sum_{i=1}^{n} \xi_i + \sum_{i=1}^{n-1} \left((T+\theta) - \left\{ \frac{\xi_i}{T+\theta} \right\} (T+\theta) \right) + \sum_{i=1}^{n-1} \eta_i,$$

$$U_n = \sum_{i=1}^{n} \xi_i + \sum_{i=1}^{n} \left((T+\theta) - \left\{ \frac{\xi_i}{T+\theta} \right\} (T+\theta) \right) + \sum_{i=1}^{n} \eta_i.$$

where $\{x\}$ is a fractional part of x. Taking into account the failure condition, we obtain the probability of a failure during a renewal interval:

$$q = \sum_{n=1}^{\infty} \int_0^{\infty} M \left(I_{U_n \le x < V_n - \alpha} I_{\Delta_n > 0} + \sum_{i=1}^{\left[\frac{\xi_n}{T+\theta}\right]} I_{V_{n-1}+(i-1)(T+\theta)+T \le x < V_{n-1}+i(T+\theta)-\alpha} I_{\zeta>0} \right) dF_\chi(x)$$

where $\Delta_n = \eta_n + \sigma_n - \alpha$, $\sigma_n = T + \theta - \left\{ \frac{\xi_n}{T+\theta} \right\} (T+\theta)$, $\zeta = \theta - \alpha$ and I_A is an indicator function of the event A. The application of renewal limit theorems (Rausand and Høyland 2004) yields

$$q \approx \frac{1}{M\eta + (T+\theta) + (T+\theta)M\left[\frac{\xi}{T+\theta}\right]} \left(\int_0^{\infty} y dF_\Delta(y) + M\left[\frac{\xi}{T+\theta}\right] \int_0^{\infty} y dF_\zeta(y) \right).$$

The Monte-Carlo method can be used to estimate $\int_0^{\infty} y dF_\Delta(y)$ and $\int_0^{\infty} y dF_\zeta(y)$.

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1} (\chi_i + \beta_i + \gamma_i) + \chi_\nu + \alpha,$$

where

$$P(\nu = n) = q(1-q)^{(n-1)}$$

and

$$0 \le \beta_i < \alpha.$$

We obviously have

$$F_\omega(t) = P(\omega \le t) = P\left(\sum_{i=1}^{\nu-1} (\chi_i + \beta_i + \gamma_i) + \chi_\nu + \alpha \le t \right). \qquad (1)$$

Applying the Laplace– Stieltjes transform to (1), we obtain

$$\tilde{F}_\omega(s) = \frac{q\tilde{F}_\alpha(s)\tilde{F}_\chi(s)}{1-(1-q)\tilde{F}_\chi(s)\tilde{F}_\beta(s)\tilde{F}_\gamma(s)}. \qquad (2)$$

Taking into account (2), we obtain

$$M\omega = M\chi + M\alpha + \frac{1-q}{q}(M\chi + M\beta + M\gamma).$$

Using stochastic ordering (Stoyan 1983), we get following estimations

$$M\chi + M\alpha + \frac{1-q}{q}(M\chi + M\gamma) \le M\omega \le M\chi + M\alpha + \frac{1-q}{q}(M\chi + M\alpha + M\gamma),$$

$$L^{-1}\left[\frac{q\tilde{F}_\alpha(s)\tilde{F}_\chi(s)}{1-(1-q)\tilde{F}_\chi(s)\tilde{F}_\alpha(s)\tilde{F}_\gamma(s)}\right] \le F_\omega(t) \le L^{-1}\left[\frac{q\tilde{F}_\alpha(s)\tilde{F}_\chi(s)}{1-(1-q)\tilde{F}_\chi(s)\tilde{F}_\gamma(s)}\right]$$

where $L^{-1}[\tilde{f}(s)]$ is an inverse Laplace– Stieltjes of $\tilde{f}(s)$.

Consider now the following trivial example. Suppose $F_\chi(t) = 1 - e^{-\lambda_\chi t}$, $F_\gamma(t) = 1 - e^{-\lambda_\gamma t}$, $F_\xi(t) = 1 - e^{-\lambda_\xi t}$, $F_\eta(t) = 1 - e^{-\lambda_\eta t}$, $F_\alpha(t) = 1 - e^{-\lambda_\alpha t}$. Therefore

$$M\left[\frac{\xi}{T+\theta}\right] = \frac{e^{-\lambda_\xi(T+\theta)}}{1-e^{-\lambda_\xi(T+\theta)}},$$

$$q \approx \frac{1}{\frac{1}{\lambda_\eta} + (T+\theta) + (T+\theta)M\left[\frac{\xi}{T+\theta}\right]}\left(\int_0^\infty ydF_\Delta(y) + M\left[\frac{\xi}{T+\theta}\right]\int_0^\infty ydF_\zeta(y)\right),$$

$$\frac{1}{\lambda_\chi} + \frac{1}{\lambda_\alpha} + \frac{1-q}{q}\left(\frac{1}{\lambda_\chi} + \frac{1}{\lambda_\gamma}\right) \le M\omega \le \frac{1}{\lambda_\chi} + \frac{1}{\lambda_\alpha} + \frac{1-q}{q}\left(\frac{1}{\lambda_\chi} + \frac{1}{\lambda_\alpha} + \frac{1}{\lambda_\gamma}\right).$$

The Monte-Carlo method can be used to estimate $\int_0^\infty ydF_\Delta(y)$ and $\int_0^\infty ydF_\zeta(y)$.

4 Conclusion.

A new reliability model for a safety system-protected object complex is presented in this paper. The proposed model can be easily used for different types of objects like nuclear power plants and others. This method can be used to estimate such reliability indices as mean time to failure and probability of failure prior to time t.

References

Gnedenko, B. and I. Ushakov (1995). *Probabilistic Reliability Engineering.* John Wiley & Sons, Inc.

Pereguda, A. I. (2001). Calculation of the reliability indicators of the system protected object-control and protection system. *Atomic Energy 90*(6), 460–468.

Rausand, M. and A. Høyland (2004). *System Reliability Theory: Models, Statistical Methods and Applications* (Second ed.). New York: John Wiley & Sons, Inc.

Stoyan, D. (1983). *Comparison Methods for Queues and Other Stochastic Models.* New-York: Wiley-Interscience.