A reliability model for safety system-protected object complex.

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Abstract

In this paper an advanced safety system-protected object complex reliability model has been proposed, assuming the safety system has a complex structure. The model can be used to estimate such reliability indices as mean time to failure and probability of failure prior to time t.

1 Introduction

There are a lot of methods available for the estimation of reliability indices of complex systems (Birolini 2002): key item method, successful path method, state space method, boolean function method, Markov and semi-Markov models. In order to estimate the reliability of a safety system-protected object complex it is crucial to consider an order of failures. Only Markov and semi-Markov models are well-suited for this task. Unfortunately, for large systems the use of Markov and semi-Markov models can become time consuming because of the large number of states involved. For this reason Pereguda (Pereguda 2001) developed a new model based on the superposition of alternating renewal processes. In the present study we set out to analyze the reliability of a safety system-protected object complex, assuming the safety system has a complex structure. We follow Pereguda (2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. Our objective is to provide an asymptotic estimation for such reliability indices as mean time to failure and probability of failure prior to time t.

2 Problem formulation

Consider a safety system that consists of N subsystems. Let the j-th subsystem consists of M_j elements. For each safety subsystem there exists the corresponding protected object failure type. Let safety system elements as well as protected object are repairable. Let items are restored to an as-good-as-new state. Consider two types of safety system failures: hidden failures and false failures. It is assumed that hidden failures can be detected only during preventive maintenance. All failures are supposed to be independent. By $\chi_{j,1}$ denote the time to the first protected object failure of j-th type. By $\chi_{j,i}$, $i = 2, 3, \ldots$ denote the time to the protected object failure of the j-th type after the $(i-1)$-th repair. Let $\chi_{j,i}$, $i = 1, 2, \ldots$ be independent and identically distributed (i.i.d.) random variables for every fixed j. By $\gamma_{j,i}$ denote the time to the i-th repair after the failure of the j-th type. Let $\gamma_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random variables for every fixed j. Suppose $\gamma_{j,i}$, $j = 1, 2, \ldots, N$, $i = 1, 2, \ldots$ are renewal points of the operation process of the complex. By $\xi_{k,j,1}$ denote the time to the first hidden failure of the k-th element of the j-th safety subsystem. By $\xi_{k,j,i}$, $i = 2, 3, \ldots$ denote the time to the hidden failure of the k-th element of the j-th safety subsystem after the $(i-1)$-th repair. Let $\xi_{k,j,i}$, $i = 1, 2, \ldots$ be i.i.d. random variables for every fixed j and k. By $\xi_{j,1}(\xi_{1,j,1}, \ldots, \xi_{M_j,j,1})$ denote the time to the first hidden failure of the j-th safety subsystem. By $\xi_{j,i}(\xi_{1,j,i}, \ldots, \xi_{M_j,j,i})$, $i = 2, 3, \ldots$ denote the time to the hidden failure of the j-th safety subsystem after the $(i-1)$-th repair. By $\eta_{j,i}$ denote the time to the i-th repair of the j-th safety subsystem after a hidden failure. Let $\eta_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random variables for every fixed j. Suppose $\eta_{j,i}$, $i = 1, 2, \ldots$ are renewal points of the operation process of the j-th safety subsystem. Let the preventive maintenance of the j-th safety subsystem is performed at periodic time intervals T_j. Let the duration of the preventive maintenance of the j-th safety subsystem is θ_j. By $\phi_{k,j,1}$ denote the time to the first false failure of the k-th element of the j-th safety subsystem. By $\phi_{k,j,i}, i = 2, 3, \ldots$ denote the time to the false failure of the

k-th element of the j-th safety subsystem after $(i-1)$-th repair. Let $\phi_{k,j,i}$, $i = 1, 2, \ldots$ be i.i.d. random variables for every fixed j and k. By $\phi_{j,1}(\phi_{1,j,1}, \ldots, \phi_{M_j,j,1})$ denote the time to the first false failure of the j-th safety subsystem. By $\phi_{j,i}(\phi_{1,j,i}, \ldots, \phi_{M_j,j,i})$, $i = 2, 3, \ldots$ denote the time to the false failure of the j-th safety subsystem after the $(i-1)$-th repair. By $\psi_{j,i}$ denote the time to the i-th repair after the false failure of the j-th safety subsystem. Let $\psi_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random variables for every fixed j. Suppose $\psi_{j,i}$, $j = 1, 2, \ldots, N$, $i = 1, 2, \ldots$ are renewal points of the operation process of the complex. By ω denote the time to the first failure of the safety system-protected object complex. Our aim is to estimate $E[\omega]$ and $F_\omega(t) = Pr\{\omega \le t\}$.

3 Main results

Since the operation process of the complex is a superposition of alternating renewal processes, it follows that

$$\omega = \sum_{i=1}^{\nu-1} \tau_i + \tau'_\nu$$

where

$$\begin{aligned}
\tau_i = (\chi_{1,i} + \gamma_{1,i}) I_{\chi_{1,i} < min\left(\chi_{2,i}, \ldots, \chi_{N,i}, \phi_1\left(\phi_{1,1,i}, \ldots, \phi_{M_1,1,i}\right), \ldots, \phi_N\left(\phi_{1,N,i}, \ldots, \phi_{M_N,N,i}\right)\right)} + \cdots \\
+ (\chi_{N,i} + \gamma_{N,i}) I_{\chi_{N,i} < min\left(\chi_{1,i}, \ldots, \chi_{N-1,i}, \phi_1\left(\phi_{1,1,i}, \ldots, \phi_{M_1,1,i}\right), \ldots, \phi_N\left(\phi_{1,N,i}, \ldots, \phi_{M_N,N,i}\right)\right)} \\
+ \left(\phi_1\left(\phi_{1,1,i}, \ldots, \phi_{M_1,1,i}\right) + \psi_{1,i}\right) \\
\times I_{\phi_1\left(\phi_{1,1,i}, \ldots, \phi_{M_1,1,i}\right) < min\left(\chi_{1,i}, \ldots, \chi_{N,i}, \phi_2\left(\phi_{1,2,i}, \ldots, \phi_{M_2,2,i}\right), \ldots, \phi_N\left(\phi_{1,N,i}, \ldots, \phi_{M_N,N,i}\right)\right)} + \cdots \\
+ \left(\phi_N\left(\phi_{1,N,i}, \ldots, \phi_{M_N,N,i}\right) + \psi_{N,i}\right) \\
\times I_{\phi_N\left(\phi_{1,N,i}, \ldots, \phi_{M_N,N,i}\right) < min\left(\chi_{1,i}, \ldots, \chi_{N,i}, \phi_1\left(\phi_{1,1,i}, \ldots, \phi_{M_1,1,i}\right), \ldots, \phi_{N-1}\left(\phi_{1,N-1,i}, \ldots, \phi_{M_{N-1},N-1,i}\right)\right)},
\end{aligned} \tag{1}$$

$$\tau'_i = min(\chi_{1,i}, \ldots, \chi_{N,i}),$$

$$Pr\{\nu = n\} = (1-r)^{n-1} r,$$

r is the probability of a failure during a renewal interval and I_A is an indicator function of the event A. The minimal cuts and paths method (Barlow and Proschan 1975) is used to find $\phi_j\left(\phi_{1,j,i}, \ldots, \phi_{M_j,j,i}\right)$. Since τ_i are i.i.d. random variables, it follows that

$$E[\omega] = E[\tau'] + \frac{1-r}{r} E[\tau].$$

Taking into account (1), we obtain

$$\begin{aligned}
E[\tau] = \int_0^\infty \left(\prod_{j=1}^N (1 - F_{\chi_j}(t)) \prod_{j=1}^N \left(1 - F_{\phi_j(\phi_{1,j}, \ldots, \phi_{M_j,j})}(t)\right) \right) dt \\
+ \sum_{j=1}^N E[\gamma_j] \int_0^\infty \left(\prod_{\substack{l=1 \\ l \ne j}}^N (1 - F_{\chi_l}(t)) \prod_{l=1}^N \left(1 - F_{\phi_l(\phi_{1,l}, \ldots, \phi_{M_l,l})}(t)\right) \right) dF_{\chi_j}(t) \\
+ \sum_{j=1}^N E[\psi_j] \int_0^\infty \left(\prod_{l=1}^N (1 - F_{\chi_l}(t)) \prod_{\substack{l=1 \\ l \ne j}}^N \left(1 - F_{\phi_l(\phi_{1,l}, \ldots, \phi_{M_l,l})}(t)\right) \right) dF_{\phi_j(\phi_{1,j}, \ldots, \phi_{M_j,j})}(t).
\end{aligned}$$

We obviously have

$$E[\tau'] = \int_0^\infty \left(\prod_{j=1}^N (1 - F_{\chi_j}(t)) \right) dt.$$

It is clear that

$$r = Pr\left\{min(\chi_1,\ldots,\chi_n) \leqslant min(\varphi_1(\varphi_{1,1},\ldots,\varphi_{M_1,1}),\ldots,\varphi_N(\varphi_{1,N},\ldots,\varphi_{M_N,N}))\right\}$$
$$\times \sum_{j=1}^{N} q_j Pr\{\chi_j < min(\chi_1,\ldots,\chi_{j-1},\chi_{j+1},\ldots,\chi_N)\}$$

where q_j is the probability of the j-th safety subsystem hidden failure during a renewal interval. Therefore, we have

$$r = \int_0^\infty \prod_{j=1}^{N} (1 - F_{\varphi_j(\varphi_{1,j},\ldots,\varphi_{M_j,j})}(t))dF_{min(\chi_1,\ldots,\chi_N)}(t) \times \sum_{j=1}^{N} q_j \int_0^\infty \prod_{\substack{l=1\\ l\neq j}}^{N} (1 - F_{\chi_l}(t))dF_{\chi_j}(t)$$

where

$$F_{min(\chi_1,\ldots,\chi_N)}(t) = 1 - \prod_{j=1}^{N} (1 - F_{\chi_j}(t)).$$

By $P_j^+(t)$ denote the probability that the j-th safety subsystem is not failed at time t. By $\langle x\rangle$ denote the integer part of x. We have

$$P_j^+(t) = f_j(t) + \int_0^t P_j^+(t-z)dF_{\tau_{ss\,j}}(z)$$

where

$$f_j(t) = (1 - F_{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}(t)) - \sum_{m=1}^{\infty}\left(1 - F_{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}(m(T_j+\theta_j))\right)\left(I_{(m-1)(T_j+\theta_j)+T_j \le t} - I_{m(T_j+\theta_j)}\right)$$

and

$$F_{\tau_{ss\,j}}(z) = Pr\left\{\left(\left\langle \frac{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}{T_j+\theta_j}\right\rangle + 1\right)(T_j+\theta_j) + \eta_j \le z\right\}.$$

The minimal cuts and paths method (Barlow and Proschan 1975) is used to find $\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})$. Finally, we obtain

$$q_j \approx \lim_{t\to\infty}(1 - P_j^+(t)) = 1 - \frac{E[\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})] - \theta_j E\left[\left\langle \frac{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}{T_j+\theta_j}\right\rangle\right]}{E[\eta_j] + (T_j+\theta_j) + (T_j+\theta_j)E\left[\left\langle \frac{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}{T_j+\theta_j}\right\rangle\right]}.$$

Moreover, we use the limit theorem from (Barzilovich et al. 1983). If

$$\frac{E[\tau^2]r}{(E[\tau])^2} \to 0,$$

then

$$F_\omega(t) \approx 1 - \exp\left[-\frac{rt}{E[\tau]}\right].$$

The Monte-Carlo method can be used to estimate $E[\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})]$, $E\left[\left\langle \frac{\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})}{T_j+\theta_j}\right\rangle\right]$ and $F_{\phi_j(\phi_{1,j},\ldots,\phi_{M_j,j})}(t)$.

4 Conclusion.

A new reliability model for a safety system-protected object complex is presented in this paper. The proposed model can be easily used for different types of objects like nuclear power plants and others. Our method can be used to estimate such reliability indices as mean time to failure and probability of failure prior to time t. This model is more computationally efficient than Markov and semi-Markov models especially for large systems.

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Abstract

In this paper an advanced safety system-protected object complex fuzzy reliability model has been proposed, assuming the safety system has a complex structure. The model can be used to estimate mean time to failure of the complex.

1 Introduction

In this paper we propose a new safety system-protected object complex reliability model, assuming the reliability indices uncertainty of a random fuzzy type. We follow the random fuzzy theory presented by Liu (2002). We follow Pereguda (2001) in assuming that the operation of the complex can be described using a superposition of alternating renewal processes. We make use of an expected value operator of a random fuzzy variable introduced by Liu and Liu (2003). Our objective is to provide an asymptotic estimation for mean time to failure of the complex.

2 Problem formulation

Consider a safety system that consists of N subsystems. Let the j-th subsystem consists of M_j elements. For each safety subsystem there exists the corresponding protected object failure type. Let safety system elements as well as protected object are repairable. Let items are restored to an as-good-as-new state. Consider two types of safety system failures: hidden failures and false failures. It is assumed that hidden failures can be detected only during preventive maintenance. All failures are supposed to be independent. By $\chi_{j,1}$ denote the time to the first protected object failure of j-th type. By $\chi_{j,i}$, $i = 2, 3, \ldots$ denote the time to the protected object failure of the j-th type after the $(i-1)$-th repair. Let $\chi_{j,i}$, $i = 1, 2, \ldots$ be independent and identically distributed (i.i.d.) random fuzzy variables for every fixed j. By $\gamma_{j,i}$ denote the time to the i-th repair after the failure of the j-th type. Let $\gamma_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random fuzzy variables for every fixed j. Suppose $\gamma_{j,i}$, $j = 1, 2, \ldots, N$, $i = 1, 2, \ldots$ are renewal points of the operation process of the complex. By $\xi_{k,j,1}$ denote the time to the first hidden failure of the k-th element of the j-th safety subsystem. By $\xi_{k,j,i}$, $i = 2, 3, \ldots$ denote the time to the hidden failure of the k-th element of the j-th safety subsystem after the $(i-1)$-th repair. Let $\xi_{k,j,i}$, $i = 1, 2, \ldots$ be i.i.d. random fuzzy variables for every fixed j and k. By $\xi_{j,1}(\xi_{1,j,1}, \ldots, \xi_{M_j,j,1})$ denote the time to the first hidden failure of the j-th safety subsystem. By $\xi_{j,i}(\xi_{1,j,i}, \ldots, \xi_{M_j,j,i})$, $i = 2, 3, \ldots$ denote the time to the hidden failure of the j-th safety subsystem after the $(i-1)$-th repair. By $\eta_{j,i}$ denote the time to the i-th repair of the j-th safety subsystem after a hidden failure. Let $\eta_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random fuzzy variables for every fixed j. Suppose $\eta_{j,i}$, $i = 1, 2, \ldots$ are renewal points of the operation process of the j-th safety subsystem. Let the preventive maintenance of the j-th safety subsystem is performed at periodic time intervals T_j. Let the duration of the preventive maintenance of the j-th safety subsystem is δ_j. By $\phi_{j,1}$ denote the time to the first false failure of the j-th safety subsystem. By $\phi_{j,i}, i = 2, 3, \ldots$ denote the time to the false failure of the j-th safety subsystem after $(i-1)$-th repair. Let $\phi_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random fuzzy variables for every fixed j. By $\psi_{j,i}$ denote the time to the i-th repair after the false failure of the j-th safety subsystem. Let $\psi_{j,i}$, $i = 1, 2, \ldots$ be i.i.d. random fuzzy variables for every fixed j. Suppose $\psi_{j,i}$, $j = 1, 2, \ldots, N$, $i = 1, 2, \ldots$ are renewal points of the operation process of the complex. By ν denote the number of renewal intervals before failure of the complex. Let ν be a integer random fuzzy variable. By ω denote the time to the first failure of the safety system-protected object complex. Our aim is to estimate $E[\omega]$.

3 Main results

Since the operation process of the complex is a superposition of random fuzzy alternating renewal processes, it follows that

$$\begin{aligned} E[\omega] &= E[\min(\chi_1,\ldots,\chi_N)] + E[\min(\chi_1,\ldots,\chi_N,\phi_1,\ldots,\phi_N)(\nu-1)] \\ &+ \sum_{j=1}^{N} E[\gamma_j I_{\chi_j \le min(\chi_1,\ldots,\chi_{j-1},\chi_{j+1},\ldots,\chi_N,\phi_1,\ldots,\phi_N)}(\nu-1)] \\ &+ \sum_{j=1}^{N} E[\psi_j I_{\phi_j \leqslant min(\chi_1,\ldots,\chi_N,\phi_1,\ldots,\phi_{j-1},\phi_{j+1}\ldots,\phi_N)}(\nu-1)]. \end{aligned} \tag{1}$$

where I_A is an indicator function of the event A. Using the definition of an expected value operator of a random fuzzy variable, we obtain

$$E[\min(\chi_1,\ldots,\chi_N)] = \int_0^\infty Cr\{\theta\in\Theta \mid E[\min(\chi_1(\theta),\ldots,\chi_N(\theta))] \ge r\}dr,$$

$$E[\min(\chi_1,\ldots,\chi_N,\phi_1,\ldots,\phi_N)(\nu-1)]$$
$$= \int_0^\infty Cr\{\theta\in\Theta \mid E[\min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))(\nu(\theta)-1)] \ge r\}dr,$$

$$E[\gamma_j I_{\chi_j \le min(\chi_1,\ldots,\chi_{j-1},\chi_{j+1},\ldots,\chi_N,\phi_1,\ldots,\phi_N)}(\nu-1)]$$
$$= \int_0^\infty Cr\{\theta\in\Theta \mid E[\gamma_j(\theta) I_{\chi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_{j-1}(\theta),\chi_{j+1}(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))}(\nu(\theta)-1)] \ge r\}dr,$$

$$E[\psi_j I_{\phi_j \leqslant min(\chi_1,\ldots,\chi_N,\phi_1,\ldots,\phi_{j-1},\phi_{j+1}\ldots,\phi_N)}(\nu-1)]$$
$$= \int_0^\infty Cr\{\theta\in\Theta \mid E[\psi_j(\theta) I_{\phi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_{j-1}(\theta),\phi_{j+1}(\theta)\ldots,\phi_N(\theta))}(\nu(\theta)-1)] \ge r\}dr.$$

Let $\chi_{j,i}$ be an exponentially distributed random variable with a mean $1/\lambda_{\chi_{j,i}}$. Suppose the value of $\lambda_{\chi_{j,i}}$ is represented as a fuzzy variable defined on the credibility space $(\Theta, \mathcal{P}(\Theta), Cr)$. Let $\mu_{\lambda_{\chi_{j,i}}}(y)$ be membership function of $\lambda_{\chi_{j,i}}$ Then $\chi_{j,i}$ is a random fuzzy variable. In the same way we define $\gamma_{j,i}$, $\xi_{k,j,1}$, $\eta_{j,i}$, $\phi_{j,i}$, $\psi_{j,i}$. Taking into account (1), we obtain

$$E[\min(\chi_1(\theta),\ldots,\chi_N(\theta))] = \frac{1}{\sum\limits_{j=1}^{N}\lambda_{\chi_j}(\theta)},$$

$$E[\min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))(\nu(\theta)-1)] = \frac{\sum\limits_{j=1}^{N}(\lambda_{\phi_j}(\theta)+(1-q_j(\theta))\lambda_{\chi_j}(\theta))}{\sum\limits_{j=1}^{N}(\lambda_{\chi_j}(\theta)+\lambda_{\phi_j}(\theta))\sum\limits_{j=1}^{N}q_j(\theta)\lambda_{\chi_j}(\theta)},$$

$$E[\psi_j(\theta) I_{\phi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_{j-1}(\theta),\phi_{j+1}(\theta)\ldots,\phi_N(\theta))}(\nu(\theta)-1)]$$
$$= \frac{\lambda_{\phi_j}(\theta)\sum\limits_{k=1}^{N}(\lambda_{\phi_k}(\theta)+(1-q_k(\theta))\lambda_{\chi_k}(\theta))}{\lambda_{\psi_j}(\theta)\sum\limits_{k=1}^{N}(\lambda_{\chi_k}(\theta)+\lambda_{\phi_k}(\theta))\sum\limits_{k=1}^{N}q_k(\theta)\lambda_{\chi_k}(\theta)},$$

$$E[\gamma_j(\theta) I_{\chi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_{j-1}(\theta),\chi_{j+1}(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))}(\nu(\theta)-1)]$$
$$= \frac{\lambda_{\chi_j}(\theta) \sum_{k=1}^{N} (\lambda_{\phi_k}(\theta) + (1-q_k(\theta))\lambda_{\chi_k}(\theta))}{\lambda_{\gamma_j}(\theta) \sum_{k=1}^{N} (\lambda_{\chi_k}(\theta) + \lambda_{\phi_k}(\theta)) \sum_{k=1}^{N} q_k(\theta)\lambda_{\chi_k}(\theta)},$$

where

$$q_j(\theta) \approx 1 - \frac{E[\xi_j(\xi_{1,j}(\theta),\ldots,\xi_{M_j,j}(\theta))] - \delta_j E\left[\left\langle \frac{\xi_j(\xi_{1,j}(\theta),\ldots,\xi_{M_j,j}(\theta))}{T_j+\delta_j} \right\rangle\right]}{E[\eta_j(\theta)] + (T_j+\delta_j) + (T_j+\delta_j) E\left[\left\langle \frac{\xi_j(\xi_{1,j}(\theta),\ldots,\xi_{M_j,j}(\theta))}{T_j+\delta_j} \right\rangle\right]}.$$

By $\langle x \rangle$ we denote the integer part of x. The minimal cuts and paths method (Barlow and Proschan 1975) is used to find $\xi_j(\xi_{1,j},\ldots,\xi_{M_j,j})$. We use credibility inversion theorem (Liu 2004) and extension principle of Zadeh (1965) to obtain

$$Cr\{\theta \in \Theta \mid E[\min(\chi_1(\theta),\ldots,\chi_N(\theta))] \ge r\},$$
$$Cr\{\theta \in \Theta \mid E[\min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))(\nu(\theta)-1)] \ge r\},$$
$$Cr\{\theta \in \Theta \mid E[\gamma_j(\theta) I_{\chi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_{j-1}(\theta),\chi_{j+1}(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_N(\theta))}(\nu(\theta)-1)] \ge r\}$$

and

$$Cr\{\theta \in \Theta \mid E[\psi_j(\theta) I_{\phi_j(\theta) \le min(\chi_1(\theta),\ldots,\chi_N(\theta),\phi_1(\theta),\ldots,\phi_{j-1}(\theta),\phi_{j+1}(\theta)\ldots,\phi_N(\theta))}(\nu(\theta)-1)] \ge r\}.$$

For example,

$$Cr\{\theta \in \Theta \mid E[\min(\chi_1(\theta),\ldots,\chi_N(\theta))] \ge r\} = \frac{1}{2}\left(\sup_{y \ge r} \mu(y) + 1 - \sup_{y<r} \mu(y)\right),$$
$$\mu(y) = \sup_{y=\frac{1}{\sum_{j=1}^{N} y_j}} \min_{1 \le j \le N} \mu_{\lambda_{\chi_j}}(y_j). \qquad (2)$$

Transformation method (Hanss 2002) is used to estimate (2).

4 Conclusion.

A new fuzzy reliability model for a safety system-protected object complex is presented in this paper. The proposed model can be easily used for different types of objects like nuclear power plants and others. Our method can be used to estimate mean time to failure of the complex, assuming the reliability indices uncertainty of a random fuzzy type.

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